IMP 113: 2nd test (Union College: Spring 2010)

Instructions:
1. Read all directions.
2. In keeping with the Union College policy on academic honesty, you should neither accept nor provide unauthorized assistance in the completion of this work.

Name: __________________________ 
Date: _________________

Direction I: Circle the correct answer

1. How much charge is stored in a capacitor, whose capacitance C = 2µF, connected to a 12V battery?
   a. 2µC  b. 12µC  c. 24µC  d. 6µC  e. 0.14µC

2. Two resistors with resistance R1 and R2 are connected in series. If R1 < R2, what can we conclude about the equivalent resistance of the two resistors, Req?
   a. R2 < Req  b. R1 < Req < R2  c. Req < R1  d. none of the above.

3. The SI unit/s for the time constant in an RC circuit, \( \tau = RC \), is/are

4. The total electrical flux through a closed surface, Gaussian surface, ________.
   a. is equal to the volume integral of divergence of the electric field.
   b. is equal to the net charge enclosed divided by the permittivity of the medium.
   c. is independent of the shape of the Gaussian surface.
   d. all of the above.
   e. none of the above

5. Which one of the following is true about the work done by an electrical force on a charge?
   a. It is path independent.
   b. It is due to change in potential energy.
   c. It is zero over a closed loop.
   d. all of the above.
   e. none of the above.

6. Which one of the following is true for a conductor during electrostatics (all charges in equilibrium)?
   a. Excess charges are evenly distributed throughout the body of the conductor.
   b. There is a net non zero electric field inside the conductor.
   c. Not every point on the conductor is an equipotential.
   d. all of the above.
   e. none of the above.

7. The node rule is an application of
   a. conservation of momentum.
   b. conservation of energy.
   c. conservation of charge.
   d. all of the above.
   e. none of the above.
Direction II: Solve the following Problems. In order to get full credit show all your work and justify your answer by reasoning.

8. Suppose that electric charge is arranged in such a way that the electric field, \( \mathbf{E} \), is given by

\[ \mathbf{E}(x, y, z) = y\mathbf{i} + 2z\mathbf{j} + z^2\mathbf{k}, \]

where \( \mathbf{E} \) is measured in \( \text{N/C} \) and the coordinates are measured in meters.

a. Let \( S_0 \) be that portion of the paraboloid \( z = 3x^2 + 3y^2 \) that is below the plane \( z=12 \), oriented by downward normals (i.e., normals with negative \( \mathbf{k} \) components). Compute the electric flux across \( S_0 \). (Compute this value directly by evaluating a certain surface integral.)
b. Let $S$ be the surface of the solid bounded by the paraboloid $z = 3x^2 + 3y^2$ and the plane $z=12$. Compute the electric flux across $S$ using the divergence theorem.

c. Compute the net charge enclosed by $S$, if the electrical permittivity in $S$ is $\varepsilon_0 = 8.854 \times 10^{-12}$ C$^2$/N·m$^2$.

d. Let $S_1$ be the part of the plane $z=12$ that is inside the paraboloid $z = 3x^2 + 3y^2$, oriented by upward normals. Use your answers to parts a and b to find the electric flux across $S_1$. Do not find the answer by computing a surface integral.
9. Refer to the matrix and system of equations below.

a. Compute the inverse of the following matrix:

\[
\begin{bmatrix}
1 & 0 & -3 \\
0 & 2 & 1 \\
1 & 1 & -4 \\
\end{bmatrix}
\]

b. Represent the following system of equations as a matrix equation, and use your answer to part a to solve it:

\[
\begin{align*}
x - 3z &= 6 \\
2y + z &= 1 \\
x + y - 4z &= -4
\end{align*}
\]

10. In figure 1, three resistors, R1, R2, and R3 are connected between terminal a and b. The terminals a and b are connected to a battery of 15 V (not shown). Terminal a is connected to the positive end of the battery and b is connected to the negative end of the battery. Find,

a. the equivalent resistance between terminal a and b.

![Figure 1. Circuit component between terminal a and b](image)
#10 continues:

b. the current that flows through R1.

c. the potential drop across R2.

d. the power dissipated by R3.

11. Let  \( A = \begin{vmatrix} 1 & 0 & 3 & -1 \\ 0 & 3 & 0 & 0 \\ 2 & 5 & -1 & 2 \\ 5 & 7 & 1 & 3 \end{vmatrix} \)

   a. Compute \( |A| \), the determinant of A.
b. Consider the following system of linear equations:

\[
\begin{align*}
    w + 3y - z &= 3 \\
    3x &= 7 \\
    2w + 5x - y + 2z &= -9 \\
    5w + 7x + y + 3z &= 4 \\
\end{align*}
\]

What information does your answer to part a tell us about the number of solutions to this system? (You are not being asked to solve this system.)

12. For each of the following, suppose that you are given some system of linear equations, you have put this system into augmented matrix form, and have applied the Gauss-Jordan procedure to obtain the given matrix. Write the system of equations that corresponds to the given matrix and interpret. (If there are no solutions, just say so. If there is exactly one solution, name it. If there are infinitely many solutions, describe them as we did in class, using one or more parameters.)

a. \[
\begin{bmatrix}
    1 & 4 & 0 & 11 \\
    0 & 0 & 1 & 7 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\]

b. \[
\begin{bmatrix}
    1 & 0 & 5 & -7 & 0 & 4 \\
    0 & 1 & 0 & 2 & 0 & -7 \\
    0 & 0 & 0 & 0 & 1 & 9
\end{bmatrix}
\]
13. In figure 2, six capacitors are connected between terminal a and b. The terminals a and b are connected to a battery of 20 V (not shown). Terminal a is connected to the positive end of the battery and b is connected to the negative end of the battery.

a. Find the equivalent capacitance of the circuit element between terminal a and b shown in Figure 2.

b. How much charge is stored on the capacitor with capacitance 4.0 F?

14. If the electrical potential of a region is given by $V(x, y, z) = x^2 + y^2 + z^2$, where $V$ is measured in volts and $x$, $y$, and $z$ are coordinates measured in meters. Find the electric field in the region in terms of the coordinates ($x$, $y$, and $z$).
15. If the electric field in a region is given by \( \vec{E}(x, y, z) = 2\hat{i} + 5\hat{j} + 6\hat{k} \), where it is measured in N/C and \( x, y, \) and \( z \) are coordinates measured in meters. Find the electrical potential associated with this field in terms of the coordinates \( x, y, \) and \( z \).

16. In Figure 3, there are two power supplies and five resistors. According to the diagram, three currents flow in the three branches of the circuit. Your instructor has chosen the direction of the current flows, randomly. Write a system of equations (but do not solve) to find the currents in the figure.

![Figure 3. Circuit with two power supplies and five resistors.](image)