Math 124  
Problem Set IV  
Due: Friday, February 25, 2005

1. Fix C and $\theta$ so that $\rho_{C,\theta} \neq i$, and let $m$ be a line.  
Prove: $\sigma_m \rho_{C,\theta} \sigma_m = \rho_{C,-\theta}$ if and only if $C$ is on $m$.

2. Suppose $\gamma_1$ and $\gamma_2$ are glide reflections with axes $m_1$ and $m_2$ respectively.  
Prove:  
   a. If $m_1$ and $m_2$ are parallel, then $\gamma_1 \gamma_2$ is a translation.  
   b. If $m_1$ and $m_2$ are not parallel, then $\gamma_1 \gamma_2$ is a rotation.

3. Prove: If $\gamma$ is a glide reflection, then $\gamma = \sigma_p \sigma_n \sigma_m$, where any one of lines $m$, $n$, and $p$ can be any arbitrarily chosen line not parallel to the axis of $\gamma$.

Extra Credit: Prove: The above statement also holds if the chosen line is parallel to the axis of $\gamma$. (The proofs for $m$, $n$, and $p$ are similar. Just do one.)