Quiz 3 Solutions

1. Suppose that the electrical potential at any point \((x,y)\) (due to some collection of electric charge) is given by the function \(V(x,y) = x^2y^4\)

   a. State the relationship between \(E\) (the electric field) and \(V\).

   \[
   E = -\nabla V
   \]

   b. Find the electric field vector at the point \((2,1)\).

   \[
   \nabla V(x,y) = 2xy^4i + 4x^2y^3j \quad \text{and therefore} \quad E(2,1) = -\nabla V(2,1) = -4i - 16j
   \]

   c. Explain in a sentence or two what the direction of your answer to part b tells us about the electrical potential.

   \(E(2,1)\) points in the direction of the quickest decrease in \(V\).

   d. Specify a vector that points from \((2,1)\) in the direction of no instantaneous rate of change of \(V\). (In other words, specify a direction that is along the equipotential surface containing the point \((2,1)\).)

   \[16i - 4j\] and \(-16i + 4j\) are both correct. There are other correct answers.

2. Let \(F(x,y) = (y^3+2)i + (3xy^2-3)j\). Show that \(F\) is conservative and compute \(\int_C F \cdot ds\) where \(C\) is the path from \((1,2)\) to \((3,-1)\) shown on the board. (Hint: Consider \(\varphi(x,y) = xy^3 + 2x - 3y\).)

   \[
   \nabla \varphi = (y^3+2)i + (3xy^2-3)j = F(x,y) \quad \text{Hence,} \quad F \text{ is conservative and } \varphi \text{ is a scalar potential function for } F.
   \]

   Then, \(\int_C F \cdot ds\) is path independent and

   \[
   \int_{(1,2)}^{(3,-1)} F \cdot ds = \varphi(3,-1) - \varphi(1,2) = (-3+6+3) - (8+2-6) = 6 - 4 = 2.
   \]
3. Four equal point charges each with a charge of 5 \( \mu \)C lie at the corners of a square with sides of length 2m.

a. Find the electric potential at the center of the square.

The potential from each charge is the same and given by the point charge equation so

\[
V = \frac{1}{4\pi \varepsilon_0} \cdot 4 = 4 \times (9 \times 10^9) \times (5 \times 10^{-6}) / \sqrt{1+1} = 1.27 \times 10^5 \text{ V}
\]

The distance \( r \) is given by the distance from a corner to the center of the square.

b. How much work would it take to bring a 2 \( \mu \)C charge from far away to the center of the square?

Work equals the product of the charge that is moved times the potential it is moved through:

\[
W = q_o \Delta V = q_o (V(\text{center}) - 0) = (2 \times 10^{-6}) \times (1.27 \times 10^5) = 0.25 \text{ J}
\]

c. Find the electric field at the center of the square (state your reasoning).

The electric field at the center is zero because the four vector electric fields from each charge cancel by symmetry.