Ampere’s Law

Using the result for the magnetic field around a long straight current-carrying wire, \( B = \frac{\mu_0 I}{2\pi r} \) and the fact that B maps to circles around the wire, we can evaluate the following integral

\[ \oint B \cdot d\vec{s} , \]

where the integral is evaluated around a closed circle centered on the wire as shown.

Now, since B is a constant in magnitude on any such circle, and since B and ds are parallel along the circle, the integral can be simply evaluated as follows:

\[ \oint B \cdot d\vec{s} = \oint B ds = B\oint ds = B(2\pi r) . \]

Substituting the expression for B, above, we find that the original integral is given by

\[ \oint B \cdot d\vec{s} = \mu_0 I . \]

This result is much more generally true; it holds for any arbitrary closed curve C that encloses a constant current I, and is known as Ampere’s law. Written a bit more generally Ampere’s law is

\[ \oint_C B \cdot d\vec{s} = \mu_0 I_{\text{enclosed}} , \]

where C is an arbitrary closed curve and \( I_{\text{enclosed}} \) is the net current enclosed by the curve C.

Ampere’s law is limited to constant currents. We will generalize it shortly to allow for time-varying currents, and obtain another of the four fundamental equations known as Maxwell’s equations. In its current form, it is most useful in situations of symmetry, on the style of those configurations for which we applied Gauss’s law to find E. In this case, we will use symmetry to allow us to calculate B in certain limited cases for which we have enough symmetry so that we can avoid actually performing directly the integral in Ampere’s law. The best way to see this is to do a few examples.

**Example:** Magnetic field of a long straight wire – both outside and inside the wire.

**Example:** Magnetic field of a toroid.

**Example:** Magnetic field of a solenoid.