

Conductors in Electrostatic Equilibrium

Electrostatic equilibrium means that there is no net flow of electric charge or no electric current. Here we will study the properties of **conductors in electrostatic equilibrium**.

1. **The electric field inside a conductor is zero in electrostatic equilibrium.**

Why is this so? We'll prove this by contradiction. Suppose there was an E field inside the conductor. This would cause free charges inside the conductor to move and hence contradict the fact that they are in electrostatic equilibrium.

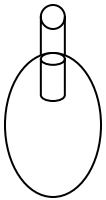
What happens when an isolated conductor is placed in an electric field? How does the internal field come to be zero? Well, at the instant the external field is applied to the conductor, the free electrons, initially uniformly distributed, migrate under the influence of the external field and produce their own electric field that cancels the external electric field. This cancellation occurs extremely quickly – within about 10^{-16} s – essentially instantaneously.

If the conductor is not isolated, but is connected to a power supply/battery, then there will be an electric field inside the wire – and it is this field that produces the sustained electric current. We will return to this in a bit.

2. **Any net charge on an isolated conductor resides on its surface.**

We can prove this using Gauss's law. Choose a gaussian surface that everywhere lies just inside the conductor surface. We just learned that the electric field must be zero everywhere on this gaussian surface so that the flux of E over this surface is zero; so by Gauss's law, there can be no internal net charge. All the net charge must therefore lie on the surface of the conductor. This statement does not tell us how the charge is distributed on the surface, but if the surface is symmetric, such as a sphere, then the net charge will distribute itself uniformly.

3. **The electric field just outside the surface of an isolated conductor is perpendicular to the surface and has a magnitude equal to σ/ϵ_0 , where σ is the local surface charge density.**



To prove this we first argue about the direction of the electric field. If it had any component parallel to the surface of the conductor, then this would cause surface charges to move, in contradiction with the assumption of electrostatic equilibrium. Therefore the electric field must lie perpendicular to the surface everywhere. Now, to get the magnitude let's use Gauss's law and choose a Gaussian surface just like that used in the planar example – a small cylinder with walls perpendicular to the surface and end caps an equal very small distance $\pm \Delta L$ from the surface. Since the electric field is zero inside and perpendicular to the conductor surface, the only contribution to the flux comes from the outer end cap of area A, so the net flux is simply EA, where E is the electric field just outside the surface, an arbitrary distance ΔL away. According to Gauss's law, this is proportional to the net charge inside the Gaussian surface, which is equal to σA , since the Gaussian cylinder intercepts the same area A on the conductor surface. Therefore, we have that $EA = \sigma A/\epsilon_0$, or $E = \sigma/\epsilon_0$.

This electric field is only constant in magnitude if σ is uniform over the conductor surface, and this only happens with sufficient symmetry.

4. **On an irregularly shaped conductor, σ , and hence E just outside, is greatest where the radius of curvature is smallest.**

We will not prove this but note that this is why the electric field is largest near sharp point of a conductor (recall the field mapping lab).

5. Every point on the surface of a conductor in electrostatic equilibrium is at the same potential – the surface is an equipotential.

This is easy to prove, since we now know that the E field is everywhere perpendicular to the conductor surface and so in the definition of

$\Delta V = -\int \vec{E} \cdot d\vec{s}$, is the line integral is confined to the surface, then the two vectors are always perpendicular to each other and $\Delta V = 0$ for any points on the surface.

Example: A solid conducting sphere with a charge $2Q$ and radius a is surrounded by a concentric conducting spherical shell of inner radius b and outer radius c carrying a charge $-Q$. Find the E field everywhere and the charge distributions everywhere when in electrostatic equilibrium.