E for continuous charge distributions: To generalize our discussion of the electric field from a set of point charges to a continuous distribution of charge, we use ideas from calculus:

$$\vec{E} = \lim_{N \to \infty} \sum_{i=1}^{N} \frac{\Delta Q_i}{r_i^2} \hat{r}_i = \frac{1}{4\pi\varepsilon_0} \int \frac{dQ}{r^2} \hat{r}$$

Note that $r (=|\vec{r}|)$ and $\hat{r}$ vary with $dq$ – they are not constants.

It is useful to express the rather abstract integration over the charges $dq$ in terms of integration over spatial variables. We can do this by introducing the notion of charge density:

$$dq = \lambda dx = \sigma dA = \rho dV,$$

which defines the linear, surface and volume charge densities. Then, depending on whether the charge is along a line, a surface, or in a volume, we can write, substituting for $dq$, that

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}) dV}{r^2} \hat{r} \text{ or } \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(\vec{r}) dA}{r^2} \hat{r} \text{ or } \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda(\vec{r}) d\ell}{r^2} \hat{r}$$

g. Example calculation for straight finite line charge (and infinite limit)

h. Second example for ring of charge

i. Example of disk of charge

j. Motion of charged particles in a uniform E field: A charge $q$ in a uniform electric field $E$ will experience a constant force $F$ given by $F = qE$ and will thus accelerate according to $qE = ma$. If the field is non-uniform, then the acceleration will vary with position (in general in both magnitude and direction) although the same equation will hold, if written as a vector equation. As an example of motion in a uniform E field suppose that a charge $q$ is released from rest at the origin and experiences this constant force. What can we say about its subsequent motion?

We know that $a = qE/m$ and using the equations of kinematics, valid for constant acceleration, then we have that:

$$x = \frac{1}{2}at^2 ; v = at ; \text{ and } v^2 = 2ax, \text{ all with } a = qE/m.$$ Further, the kinetic energy as a function of distance is given by $K = \frac{1}{2}mv^2 = max = qEx – an$
expression which can also be obtained using \( W = K = Fx = qEx \).

k. Oscilloscope/Cathode Ray Tube: Demo Let’s examine what happens if an electron enters a region between two oppositely charged parallel plates (we will see that this situation results in an approximately uniform electric field between the plates directed from + to – plate).

\[
\begin{align*}
\text{vi} & \quad \text{L} & +\
-e & \quad \phantom{\text{L}} & -
\end{align*}
\]

The acceleration of the electron is given by \( \ddot{a} = (-eE/m)\hat{j} \) (note that we ignore gravity here) and, remember, that since the only acceleration is along the –y direction, that the x-component of velocity remains unchanged. The vertical velocity is given by \( v_y = (-eE/m)t \). We can then also write the trajectory of the electron:

\[
\begin{align*}
x = vi & \quad \text{and} \quad y = \frac{1}{2}ayt^2 = -\frac{1}{2}(eE/m)t^2. \\
\text{Combining these two equations by eliminating } t, \text{ we can write that } t = x/v_i \text{ so that } y = -\frac{1}{2} (eE/m)(x/v_i)^2, \text{ representing the equation of a parabola.}
\end{align*}
\]

When the electron emerges after passing through the plates it has been steered, or deflected, to emerge traveling at a downward angle \( \theta \), given by \( \tan \theta = v_y/v_x \), where \( v_y \) is taken at the time \( t = L/v_i \) corresponding to the time the electron leaves the plates.

\[
\begin{align*}
v_x & \quad \theta \\
v & \quad v_y
\end{align*}
\]

We find that \( \theta = \tan^{-1}(eEL/mv_i^2) \).

In a cathode ray tube (in TVs, computer monitors – not flat screen ones – and oscilloscopes) two pairs of such plates are used to steer the electron beam both horizontally and vertically.
1. **Dipole in an Electric Field** - A dipole in a uniform electric field will not feel a net force, but will experience a torque tending to align the dipole with the electric field. The torque can be written as \( \vec{\tau} = \vec{p} \times \vec{E} \). Because of this torque, a dipole in a uniform electric field will also have a potential energy of interaction. We can calculate this (relative to zero when the dipole and field are perpendicular to each other) to be given by \( U = -\vec{p} \cdot \vec{E} \), so that this energy is a minimum when the dipole is oriented along the electric field and a maximum when it is anti-parallel to the field.