Problem Set #12
Chapter 29

2. a) No b) yes c) No using \( F = q \vec{v} \times \vec{B} \)

6. Charge must be negative, since it is attracted to plate of higher \( V - n \) plate. Using \( \vec{F} = q \vec{v} \times \vec{B} \) with \( \vec{g} < 0 \)
a) into paper b) out of paper c) out of paper d) into paper e) out of paper f) out of paper

9. a) 1 - 180° 2 - 270° 3 - (straight sections \( \Rightarrow \text{net} \vec{F} = 0 \))
    - curved section gives \( \text{Fnet at} 90° \)

4. - (again, straight sections \( \Rightarrow \text{Fnet} = 0 \))
5. - 315° or (-45°)
6 - 225° (same reason as #5)
7 - 125°
8 -

- two straight sections \( \Rightarrow \theta = 90° \) curved section \( \Rightarrow 45° \) average,

- so overall average is \( 90° \times \frac{2}{3} + 45° \times \frac{1}{3} = 60° + 15° = 75° \)

(note answer in text is wrong here -)

b) 1 = 2 > 3 = 4     g) c) 8 > 5 = 6 > 7

Problems

1. a) \( |\vec{F}| = (2 \text{ c}) v B \sin \theta = (6.2 \times 10^{-16}) (550)(0.045) \sin 52° = 6.2 \times 10^{-18} \text{ N} \)

b) \( a = \frac{F/m}{6.6 \times 10^{-24} \text{ kg}} = 9.5 \times 10^8 \text{ m/s}^2 \)

c) Since \( \vec{F} \perp \vec{v} \) this \( a \) is not real - or at least not real (\( \frac{\vec{F}}{\vec{v}} \)) is constant
4 a. \[ F = e \cdot \vec{V} \times \vec{B} = (-1.6 \times 10^{-19})(2 \times 10^6 \hat{i} + 3 \times 10^6 \hat{j}) \times (0.02 \hat{k}) - \]
\[ = (-1.6 \times 10^{-19})[-2.15 \times 10^6 - 3(0.03) \times 10^5] \hat{k} \]
\[ = 1.6 \times 10^{-19} [3 \times 10^5 + 0.9 \times 10^5] \hat{k} = 6.2 \times 10^{-14} \text{ N} \hat{k} \]

b. For a proton, the only change is the sign of the charge. So only the direction changes to \(-\hat{k}\).

17. a) \[ S \text{in} \bar{K} = \frac{q}{2} m \bar{V}^2 = 1.2 \text{ keV} \Rightarrow V = 1.2 \times 10^3 eV \]
\[ = (1.2 \times 10^3)(1.6 \times 10^{-19}) \]
\[ \text{Solving for } \bar{V} \Rightarrow \]
\[ V = \frac{2 \bar{K}}{m} = 2.05 \times 10^7 \text{ m/s} \]

b) since \[ q \bar{V} \bar{B} = m \bar{V}^2 \text{ then solving for } r \Rightarrow \]
\[ r = \frac{m \bar{V}}{q \bar{B}} = \frac{(9.11 \times 10^{-31})(2.05 \times 10^7)}{(1.6 \times 10^{-19})(0.25)} = 4.7 \times 10^{-4} \text{ m} \]

c) \[ f = \frac{\bar{V}}{2 \pi r} = \frac{2.05 \times 10^7}{2 \pi (0.25)} = 1.3 \times 10^7 \text{ Hz} \]

D. b) To find \[ \bar{V}(t) \], we need to solve the equation \[ \bar{F} = m \bar{a} \]
find \( \bar{a} \) and integrate to find \[ \bar{V}(t) \] but,
\[ \bar{F} = e \bar{V}(t) \times \bar{B} = m \bar{a} \]

writ\[ \bar{V}(t) = \bar{V} + \bar{V}(t) \hat{i} + \bar{V}(t) \hat{j} + \bar{V}(t) \hat{k} \text{ and substitute in } \]
\[ e[\bar{V}(t) \hat{i} + \bar{V}(t) \hat{j} + \bar{V}(t) \hat{k}] \times \bar{B} \hat{a} = m \bar{a} \]
Taking the cross product, we have

$$\vec{F} = e \left[ -v_y \vec{B} \hat{k} + v_x \vec{B} \hat{j} \right] = m \left[ a_x \hat{i} + a_y \hat{j} + a_z \hat{k} \right]$$

so we conclude that

$$a_x = \frac{dv_x}{dt} = 0 \quad \Rightarrow \quad v_x = \text{constant} = v_{0x}$$

$$a_y = \frac{eB v_x}{m} = \frac{dv_y}{dt} \quad \Rightarrow \quad \frac{dv_y}{dt} = \frac{eB v_x}{m}$$

$$a_z = -\frac{eB v_y}{m} = \frac{dv_z}{dt} \quad \Rightarrow \quad \frac{dv_z}{dt} = -\frac{eB v_y}{m}$$

need to solve these. This is hard.

A trick: differentiate both equations w.r.t. time.

$$\frac{d^2 v_y}{dt^2} = \frac{eB}{m} \frac{dv_z}{dt} = \frac{eB}{m} \left(-\frac{eB v_y}{m} \right)$$

$$\frac{d^2 v_z}{dt^2} = -\frac{eB}{m} \frac{dv_y}{dt} = -\frac{eB}{m} \left(\frac{eB v_x}{m} \right)$$

Then are both of the form \(\frac{d^2 v_y}{dt^2} = -Kv_y\) with \(K = \frac{eB}{m}\)

The solution is \(v_y = v_{0y} \cos(Kt)\) (check this)

So

$$v_y = v_{0y} \cos\left(\frac{eB t}{m}\right) \quad \text{(since } v_y(K = 0) = v_{0y})$$

and

Then \(\frac{dv_2}{dt} = -\frac{eB}{m} v_y\)

$$\Rightarrow \quad v_z = -v_{0y} \sin\left(\frac{eB t}{m}\right)$$

So \(\vec{V}(t) = \vec{v}_{0x} \hat{i} + v_{0y} \cos\left(\frac{eB t}{m}\right) \hat{j} - v_{0y} \sin\left(\frac{eB t}{m}\right) \hat{k}\)
29. a) If due to conservation of charge, total charge = 0
   then total charge must = 0.

   b) Due to conservation of momentum, immediately after
   the collision each will have an equal, but opposite
   momentum and so equal and opposite speeds since
   the masses are equal.

   Therefore each travels in the same radius circle
   but in opposite directions - colliding after
   \( \frac{1}{2} \) the period.

   \[ T = \frac{2\pi r}{v} \quad \text{and} \quad qvB = m \frac{v^2}{r} \quad \text{so} \quad \frac{r}{v} = \frac{mv}{qB} \]

   Therefore \( T = \frac{2\pi m}{v^2} \cdot \frac{v^2}{B} = \frac{2\pi m}{qB} \)

   The answer is \( T = \frac{\pi m}{qB} \).

35. To move this, we need an upward magnetic force equal

   \[ \begin{array}{c}
   \uparrow \quad \vec{F}_B \\
   \times \\
   \downarrow \quad \vec{mg}
   \end{array} \]

   The weight in \( F_B = (0.013 \times 9.8) \text{ N} \)

   With current to the right \( \Rightarrow \vec{F}_B \) is upward

   \[ F_B = ILB = I (0.62)(0.44) = (0.013)(9.8) \]

   \[ \Rightarrow I = 0.47 \text{ A.} \]
36. \[ \vec{F}_b = \vec{I} \times \vec{B} = (0.5\hat{i} - 0.5\hat{j}) \times (0.003\hat{j} + 0.01\hat{k}) \]
\[ = 0.5(0.5)[(0.003)\hat{k} - (0.01)\hat{j}] \]
\[ = (-2.5 \times 10^{-3}\hat{j} + 0.75 \times 10^{-3}\hat{k}) \text{ N} \]

41. If the length of wire \( L \) is made into \( N \) loops of radius \( R \)

The circumference of each loop is \( \frac{L}{N} = 2\pi R \), so the area of each loop \( A \) is:

\[ A = \pi R^2 = \pi \left( \frac{L}{2\pi N} \right)^2 = \frac{L^2}{4\pi N^2} \]

Also:

\[ \tau = \mu B = NI A B = NI \frac{L^2}{4\pi N^2} = \frac{IL^2 B}{4\pi N} \]

To minimize this, we want \( N = 1 \) \( \Rightarrow \)

\[ \tau = \frac{IL^2 B}{4\pi} \]

[Note: This is \( \tau = IAB \) where \( A = \pi R^2 = \pi \left( \frac{L}{2\pi} \right)^2 \)]

47. Due to current in loop tends to rotate cylinder uphill (turning \( A \) toward \( B \))

Weight \( mg \) helps to rotate it downhill
to balance we require the torques to be equal:

\[ NIA = B \sin \theta = mg (\sin \theta) \]

So \( NIA = mg \) or \( I = \frac{mg}{NB(L-2r)} \)

or \( I = \frac{mg}{2NB(2L)} = \frac{0.25(9.8)}{2(10)(0.5)(0.1)} = 2.45 \text{ A} \)

49a) \( \mu = \frac{IA}{NL} = \frac{2.3}{(10)(2)(0.1)} = 12.7 \text{ A} \)

b) \( \tau_{max} = \mu B = (2.3)(3.5 \times 10^{-3}) = 8.1 \times 10^{-2} \text{ N-m} \)

55a) \( \overrightarrow{F} = \overrightarrow{m} \times \overrightarrow{B} = m \left[ 0.62 - 0.8J \right] \times \left[ 0.25\hat{i} + 0.5\hat{k} \right] \)

where \( m = IA = 0.25(\pi)(0.09)^2 = 4 \times 10^{-4} \text{ A-m}^2 \)

So \( \overrightarrow{I} = m \left[ -0.18\hat{j} + 0.2\hat{k} - 0.2 \hat{i} \right] \)

\( = (-0.97 \times 10^{-4} \hat{i} - 7.2 \times 10^{-4} \hat{j} + 8 \times 10^{-4} \hat{k}) \text{ N-m} \)

b) \( \overrightarrow{U} = -\overrightarrow{m} \overrightarrow{B} \) (from text - we did not cover this in class)

Sorry - omit this part