

A VIRTUAL RECONSTRUCTION OF A VIRTUAL EXHIBIT

THOMAS F. BANCHOFF AND DAVIDE P. CERVONE

ABSTRACT. The traveling exhibit “*Para Além da Terceira Dimensão*”, now touring a number of Portuguese cities, was the result of an international cooperation between two mathematicians in the United States and a team of mathematicians and computer professionals in Portugal. Inspired by the virtual art gallery “Surfaces Beyond the Third Dimension”, itself based on a physical exhibit originally staged by the Providence Art Club in 1996, this updated version includes new images, new movies, new languages, and a new layout. We describe here the process, and some of the difficulties, of putting together such a project.

1. INTRODUCTION

On December 7, 1999, José Francisco Rodrigues, Professor of Mathematics at the University of Lisbon, visited our show “Surfaces Beyond the Third Dimension” at the Providence Art Club, and he left this message:

“Congratulations. Your exhibit is a wonderful example of how Mathematics can also be communicated through (and with) Art in the Information Society. I have really appreciated it. Hope to see more in future . . .”

The visit was virtual, and the message was prophetic. In this article, we would like to describe the collaboration that began at that point, culminating in a traveling exhibit “*Para Além da Terceira Dimensão*”. The exhibit first appeared in Óbidos, in October 2000, and the first-named author gave a presentation at the formal closing there on November 20. Two days later, both authors flew to the island of Madeira to give presentations at the opening of the show at the University of Madeira at Funchal. The next stop for the exhibit was in Coimbra, at the National Museum for Science and Technology of Portugal, and from there, it continues on to several other cities of the country. Ultimately it will visit nearly every university in Portugal.

How did this exhibit come about? What were the technical problems that had to be solved in order to transform the show to the new format? What kinds of new mathematical questions arose during the collaboration? What

new materials appeared, and what changes occurred in the old material? These are the topics that we will address in this paper.

From the beginning, we wish to thank J. F. Rodrigues and all of the colleagues in Portugal who made this virtual collaboration possible. It is hard to believe that the project took place without a single face-to-face meeting between either of the two authors and any of those working on the other side of the Atlantic. The resulting exhibit, together with its associated gallery book, CD-ROM and web site, turned out beautifully, and should prove to be a valuable illustration of how mathematics can be appealing and accessible to the non-mathematician, and still give new insight to the experienced mathematician.

2. THE ORIGINAL EXHIBIT

The Providence Art Club is a one-hundred-and-fifteen-year-old association of professional artists and supporters of the arts, currently including about five hundred members. Artist members are selected by a special committee, based on portfolios and recommendations. One of the privileges of artist membership is the chance to have a single or shared show in one of the two exhibit areas.

The first-named author became an artist member in 1979 and held his first show in 1993, featuring five large (30 by 40 inch) panels, the originals of which had been commissioned by Pepsico for their computer headquarters building. There were several other smaller panels from various projects, two large foamboard models, and a tray of computer-generated slides. There was a rudimentary exhibit booklet with titles and very brief descriptions of the mathematical meaning of the objects appearing in the exhibit. The technical aspects of image production, especially for the Pepsico series, were handled by David Margolis.

In 1996, both authors collaborated on a more elaborate exhibit, with entirely new material. There were a dozen 20 by 24 inch panels of glossy computer-generated images produced by the Ilfochrome process and mounted on foam-core. In addition, there was a television monitor continuously showing two three-minute computer animations produced by the first-named author and student assistants.

The exhibit book was more elaborate, including artists' comments about the various images, and presenting the formal mathematical descriptions and technical aspects of the computer programming that went into each image. There also was a well-attended gallery talk and personal tour of the exhibit. The Providence Journal provided a complimentary review, accompanied by an illustration from the show.

Several of the images in the exhibit came from the collaboration of the two authors on the illustrations for the first-author's *Scientific American Library* volume *Beyond the Third Dimension* [4]. One other series was due to a summer research project that prepared three panels and a videotape for the celebration of the hundredth birthday of Prof. Dirk Struik. Two other images appeared as covers of the *Notices of the American Mathematical Society*, one was the cover of the *Math Horizons* publication of the Mathematical Association of America, while another set was featured in the MAA experimental totally electronic journal *Communications in Visual Mathematics* [5].

The technical work involved in preparing the computer-generated images was handled by the second-named author while he was a post-doctoral fellow at The Geometry Center, a research institute sponsored by the National Science Foundation. It is significant to note that he never got a chance to see the actual exhibit while it was hanging in the Providence Art Club gallery during March and April!

3. THE VIRTUAL EXHIBIT

In 1997, the authors embarked on a project to construct a virtual representation of the 1996 physical exhibit as a site on the world-wide web [1]. Using photographs taken at the hanging and the opening, as well as a floor plan of the gallery, the second-named author produced a plan of the exhibit in the form of a cube viewed from above in perspective. A viewer could select any portion of a wall, which would lead them to a close-up view of that wall. See Fig. 1.



FIGURE 1. From a view of the Providence Art Club's Dodge House gallery (left), the user can click on any wall and get a close-up view of that area of the gallery (right). Clicking on an image or description card produces a page with more information about, and larger versions of, that image.

From there, the viewer can select any of the images on that wall to see a more detailed view, or can move to a view of a nearby wall. After choosing an

image, the viewer is presented with a page dedicated to that particular piece. This page includes links to a larger version of the image, a brief description of the underlying mathematics, and some bibliographic information concerning the piece. Several of the images also provide short electronic movie clips. The site included a guestbook, where visitors to the virtual gallery were invited to enter their comments. (It is this book that contains the prophetic comment of J. F. Rodrigues quoted above.)

As time permitted, we added new materials to most of the pages. Primarily, this was in the form of new animations and their descriptions. In 1998, we performed a major update by augmenting most pages with extensive discussions of the mathematics underlying the images, and included new descriptions of the images from an artistic point of view. This version of the virtual gallery became an “article” in *Communications in Visual Mathematics* [5] and this exposure generated a number of visits to the site. The process that led to the construction of this virtual exhibit is described in more detail in [3].

4. THE NEW ITINERANT EXHIBIT

Between 1998 and the fall of 2000, the virtual gallery remained relatively unchanged. People continued to visit it from around the world, and many added their comments to the guestbook. The message from J. F. Rodrigues quoted in the introduction marked the beginning of a new phase in the development of this exhibit. Soon after writing his comment, Rodrigues contacted the authors about the possibility of translating the gallery into Portuguese, and reproducing the physical exhibit in one or more cities in Portugal. We were intrigued by this suggestion, because it would both make the materials available to a wider audience, and give us the opportunity to revisit these compelling objects using modern technology, which had advanced considerably even in the four years since the original show was executed.

More than a simple reproduction of the 1996 exhibit, Rodrigues’ vision expanded the original in a number of ways. When the show was created for the Providence Art Club, we had not yet thought of the virtual gallery; but for Rodrigues, the virtual gallery *was* the exhibit, including its movie clips, textual information, and interlinkages. His version of the physical exhibit, then, would include not just the static artwork, but also several computers and a video projector displaying the electronic movies, an interactive web site dedicated to the show [2] together with computers where visitors could navigate that site while visiting the physical exhibit, and a gallery book that includes a CD-ROM containing the complete web site, so that, in a very real sense, visitors can take the show home with them. The textual information describing the artwork would be included as part of the exhibit as well, with large panels (in both Portuguese and English) below each image giving the

mathematical motivations for the piece (see Fig. 2). In hindsight, this progression from physical exhibit to virtual gallery, to combined physical and virtual experience, seems a natural one, and one that appears to have been quite successful here.

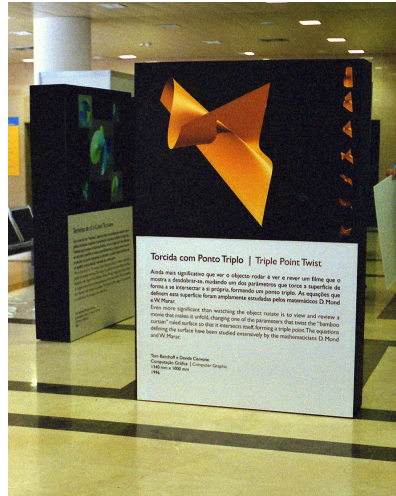


FIGURE 2. The images were mounted on free-standing frames, with text printed below them in Portuguese and English. The frames can be placed in whatever manner is appropriate for the exhibit area. Here we see the space as it appeared at the University of Madeira at Funchal.

All the elements from the virtual gallery were integrated into this new presentation of “Surfaces Beyond the Third Dimension.” Indeed, even the idea that, as a virtual gallery on the web, people from a wide geographic area can visit it has been incorporated into “*Para Além da Terceira Dimensão*”, not by having people travel to it, but rather by having it travel to them. The physical exhibit started in the medieval town of Óbidos, then moved to the modern university of Madeira at Funchal, and will be visiting a number of other cities in Portugal.

As a traveling exhibit, the images and other materials had to be made portable. The Portuguese recreation, due largely to the design of the Atelier Henrique Cayatte, accomplished this by displaying the graphics on large, free-standing frames, having on the front a computer image with its textual explanation below it, and a similar arrangement on the back. The images are printed at a much larger size than they were for the original show in 1996; here they are about a meter tall and a meter and a half wide, with rich color in a matte finish. These panels are then arranged in the gallery space so that visitors can walk in front of and behind them (see Fig. 2). The computers displaying the movies and web site are interspersed among the panels on small tables.

The current exhibit is more than just a recreation of the original one, and more even than just a physical version of the virtual art gallery that inspired it. As part of the new exhibit, Rodrigues commissioned two new works from the second-named author centered around the idea of the hypercube. This

resulted in the creation of the pieces “A Rotation of Cubes” and “Iced Cubes”, together with about a dozen new movie segments and associated text that illustrate and explain the mathematics underlying these images. A short introduction to visualizing the fourth dimension also was requested, to be added to the gallery book. This professionally designed and printed gallery book and accompanying CD-ROM, attractive works in their own right, form a valuable companion to the exhibit, and a permanent memento of the experience.

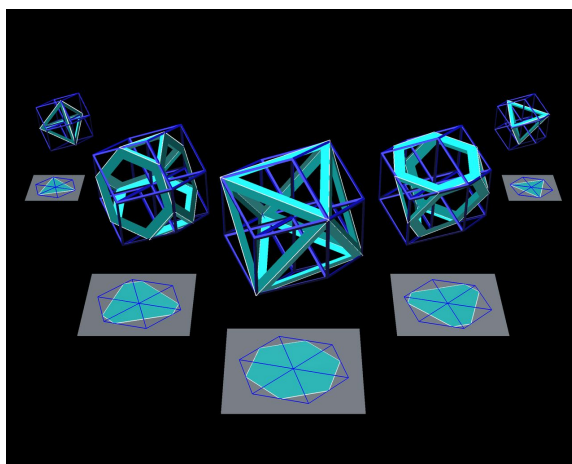


FIGURE 3. The new image “Iced Cubes” commissioned for “Para Além da Terceira Dimensão”. It shows several different slices of a four-dimensional hypercube, together with corresponding slices of a three-dimensional cube displayed in the planes below.

The production of the new materials for the exhibit in Portugal gave us the incentive to revise our own on-line virtual gallery once again. The current look of the site [1] is the result of a facelift inspired by our recent collaboration. The new movies and text that relate to the original twelve images have been added to our older materials, but the images of the hypercube and the introduction to the fourth dimension that were commissioned for the show in Óbidos are available exclusively at the Portuguese site [2].

5. CREATING THE EXHIBIT

The exhibition viewed originally by J. F. Rodrigues was a virtual one, and in many ways the collaboration that sprang from that visit was virtual as well. The entire process was carried out electronically, without a face-to-face meeting among any of the participants until after the show opened in Óbidos (see Fig. 4). This presented a number of technical problems and challenges.

The primary one involved the software used to produce the original show. Of the twelve initial images, three were produced using a custom-built program for SGI workstations written by a student (Nick Thompson) of the first-named author at Brown University more than fifteen years ago. It is remarkable that the program still runs virtually unchanged after so long a period, but the team in Lisbon did not have access to this software (or hardware) and it is not suited to the production work that they would have needed it to perform. An additional image was produced using a different custom program that is no longer available. Three others were developed using specialized software at the graphics laboratory at Brown University, and the remaining five used a very early prototype of a graphics package called **StageTools** developed by the second-named author. The two new images of the hypercube used the current version of this program.



FIGURE 4. The authors together with their host at the opening of the exhibit at Funchal. Here we see Davide Cervone (left), José Francisco Rodrigues (center), and Thomas Banchoff (right).

At the time the first virtual exhibit was designed and movie clips were added, several of the images were converted to **StageTools** format, since this is the program that could create the movies. For the new gallery, we decided to convert everything to **StageTools**. This would make it possible to produce movies of some of the objects that did not have any before, and would also mean that the Lisbon group would only need to learn one new program. In addition, **StageTools** is distributed for free [7]; it is a collection of modules for the **Geomview** 3D graphics viewer, also available for free [6]. Both programs run under the **linux** operating system, which was in use by *Arte Numérica*, the firm that was producing the materials in Portugal, so this seemed an appropriate choice.

Many of the images are made of several views of a single object (e.g., “Z-squared necklace” and the three “Tetraviews”, see Fig. 5). For the original exhibit, the individual views each were created separately, and then scaled and combined by hand to form a single image. This is not a very precise

means of handling the images, and did not seem appropriate for the long-distance collaboration that we were performing. Instead, we used some of the newer features of `StageTools` (that were unavailable in 1996) to incorporate all the views into one file automatically. This makes it possible to recreate exactly the same image as often as needed. One side effect of this process (that in hindsight should have been obvious) is that since the objects displayed by `Geomview` are in three-dimensional perspective, the ones near the edge of the images have a slightly different viewing angle than the ones near the middle. Usually this is not a problem, but in some images, where the viewing angle was sensitive to small changes, we had to compensate for it. There was also an associated bug in `Geomview` that required a work-around.

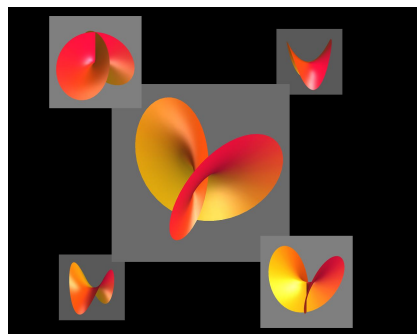


FIGURE 5. “Z-Squared Tetraview” shows one of the images that was originally produced by generating each of the five views individually, and then combining the results with grey backgrounds by hand.

One of the most difficult conversion issues involved lighting and coloration of the various surfaces. It is not hard to reproduce a given surface in `StageTools` in its basic shape, but getting it to look the same as it did in a different program can be a real challenge. Finding the right viewpoint is the first step, but after that, one has to get the lighting, color, and surface material characteristics to match. This is harder than it sounds, as there are a variety of factors that control these properties, and different programs handle them differently. Worse, the graphics systems on different hardware handle them differently as well, so what we saw on our screens here did not necessarily correspond to what the *Arte Numérica* team saw on theirs! Furthermore, because of the differences between reflected and emitted light, printed versions don’t always reproduce what we see on the screen, either. Fortunately, the final images used in the exhibit were beautiful, and fully captured the artists’ intents.

Particularly challenging was the transparency used in two of the images (“Klein bottle” and “Temple of Viviani”; see Fig. 6). The `linux` version of `Geomview` has only limited support for transparency, and it was not up to the task of reproducing these images in the quality that was required for the show. The artists in Lisbon, however, were able to use a ray-tracing program (for which `Geomview` could write the appropriate data file) to generate high-quality images that included transparency. Only moderate adjustments were

needed to the files produced from `Geomview` in order to get good results. Indeed, these images were so successful that the Klein bottle was chosen as the signature piece for the project, and it appears on all the promotional materials.

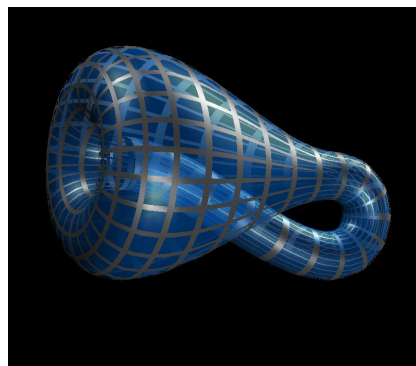


FIGURE 6. The “Klein Bottle” image included transparency effects in a fundamental way. This required the use of a ray-tracing program to produce the high-quality version used in the exhibit. The parameterization for this image was developed by Jeff Beall, a student of the first-named author at Brown University.

Certainly a major portion of the computer work for the show involved the production of the still images for display, but the electronic movies also represent a significant amount of effort. For the displays used in the traveling exhibit, *Arte Numérica* produced images at a much larger size and higher resolution than the versions available on our original web site. They are full-screen in size and each combines several of the smaller movies into one longer, repeating movie. These required considerable computation time and disk space (the final compressed movies are on the order of 70 megabytes each in length), but the results were very effective. These large-format movies are available on the CD-ROM that accompanies the gallery book, which is one of the few differences between the on-line version available at [2] and the CD version.

For the on-line gallery, we wanted the movies to be available to the widest possible audience. This meant using a file format that most people would be able to view, but that would still be small enough to transfer comfortably over slow network connections. In general, we found that the `MPEG` format produced the best quality/compression values for the kinds of images that we were using. Unfortunately, not everyone has access to an `MPEG` player. The animated `GIF` format is viewable in all the major browsers, but it has several drawbacks: it has poor compression, only supports 256 colors, and does not provide user controls for starting and stopping the animation or stepping through it. A third alternative is Apple’s `QuickTime` format, which can provide acceptable quality and compression. Plugins for `QuickTime` are available for both the Macintosh and IBM PC [8], and this player does include the frame controls that are missing from `GIF` animations. We decided to provide our movies in all three formats, so that the viewer can choose whatever format is best for his or her browser configuration. To help reduce download time for the animated `GIF` files, we reduced the image size slightly

in comparison to the other formats. Most of this work was done using `GraphicConverter` on a Macintosh computer.

The decision to use `StageTools` for all the artwork seems to have been a good one. The source files both for the movies and still images are small text files, and so could be transferred easily between the authors in the United States and the team in Portugal. Even though they were not familiar with these programs prior to this collaboration, the *Arte Numérica* group was able to modify the movie scripts on their own, and produced very high quality results. While there were a few initial startup problems, using this software was a remarkably effective method of transferring the geometric information needed for the exhibit.

6. CONCLUSION

When we finished the first virtual exhibit based on the 1996 Providence Art Club show, we felt that we had achieved our objective very well, yet we knew we had only begun to explore this new form. Subsequent developments in Internet technology opened up new possibilities for enhancing visual quality and providing additional hypertext features in the ancillary materials accompanying the images. We probably would not have gone back to redo the virtual gallery if it had not been for this opportunity to re-create the experience in a different form. J. F. Rodrigues' proposal to try something new along the same lines was the encouragement we needed to proceed with the project, in virtual collaboration on two different continents.

The experience of coming to Portugal to see the new exhibit was a thrill. The old images had new life, displayed in imaginative ways in a different sort of exhibit space. Although many of the comments and questions that arise during an exhibit are universal, the interchanges that took place at the first two sites, in Óbidos and on Madeira, gave us new insight into our own work. The most dramatic innovation is the availability of online terminals where visitors to the physical exhibit can interact directly with the geometric phenomena that are captured imperfectly in any single image or series of images. We look forward to hearing about the reactions from other places in Portugal where the traveling exhibit will appear. We hope as well to learn a good deal from reactions to the CD that accompanies the exhibit booklet. And we look forward to the next opportunity to collaborate in new incarnations of virtual art exhibits based on surfaces “beyond the third dimension”, that is, “*para além da terceira dimensão*”.

REFERENCES

- [1] Banchoff, T. F., and Cervone, D. P.: *Surfaces beyond the third dimension*, art exhibit, Providence Art Club, 31 March to 19 April, 1996; interactive electronic exhibit at [URL::http://www.math.brown.edu/~banchoff/art/PAC-9603/](http://www.math.brown.edu/~banchoff/art/PAC-9603/), 1997.
- [2] ———: *Para além da terceira dimensão*, Portuguese version of [1] produced by *Arte Numérica* under the direction of José Francisco Rodrigues; web site available at [URL::http://alem3d.obidos.org/](http://alem3d.obidos.org/).
- [3] ———: An interactive gallery on the internet: “Surfaces beyond the third dimension”, *International Journal of Shape Modeling* **5** (1999) 7–22.
- [4] Banchoff, T. F.: *Beyond the Third Dimension: Geometry, Computer Graphics, and Higher Dimensions*, Scientific American Library, W. H. Freeman and Co., New York (1990).
- [5] *Communications in Visual Mathematics*, [URL::http://www.geom.umn.edu/locate/CVM/](http://www.geom.umn.edu/locate/CVM/).
- [6] Geomview Home Page, [URL::http://www.geomview.com/](http://www.geomview.com/).
- [7] StageTools Home Page, [URL::http://www.math.union.edu/locate/StageTools](http://www.math.union.edu/locate/StageTools).
- [8] QuickTime Home Page, Apple Computer, [URL::http://quicktime.apple.com/](http://quicktime.apple.com/).