

To Prove	Do
$P \Rightarrow Q$	“Assume P is true,” prove Q is true, or “Assume Q is false,” prove P is false, or “Assume P is true and Q is false”, produce a contradiction.
$P \iff Q$	Prove $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$, or prove $(P \Rightarrow Q) \wedge (\sim P \Rightarrow \sim Q)$, or prove $(\sim Q \Rightarrow \sim P) \wedge (Q \Rightarrow P)$, or prove $(\sim Q \Rightarrow \sim P) \wedge (\sim P \Rightarrow \sim Q)$
$(\forall x)(P(x))$	“Let x be an arbitrary ...” Prove $P(x)$.
$(\exists x)(P(x))$	“Take $x = \dots$ ” Prove $P(x)$ for this x .
$A \subseteq B$	Prove $(\forall x \in A)(x \in B)$ i.e., if $x \in A$ then $x \in B$.
$A = B$	Prove $(A \subseteq B) \wedge (B \subseteq A)$.
$A = \emptyset$	Prove $(\forall x)(x \notin A)$ (frequently best to use proof by contradiction).
$x \in A \cup B$	Prove $(x \in A) \vee (x \in B)$.
$x \in A \cap B$	Prove $(x \in A) \wedge (x \in B)$.
$x \in A - B$	Prove $(x \in A) \wedge (x \notin B)$.

To Prove	Do
$\sim(P(x) \Rightarrow Q(x))$	Prove $(\exists x)(P(x) \wedge \sim Q(x))$.
$\sim(P(x) \iff Q(x))$	Prove $(\exists x)(P(x) \wedge \sim Q(x)) \vee (\exists x)(Q(x) \wedge \sim P(x))$.
$\sim(\exists x)(P(x))$	Prove $(\forall x)(\sim P(x))$.
$\sim(\forall x)(P(x))$	Prove $(\exists x)(\sim P(x))$.
$A \not\subseteq B$	Prove $(\exists x)(x \in A \wedge x \notin B)$.
$A \neq B$	Prove $(A \not\subseteq B) \vee (B \not\subseteq A)$. ie, there is an $x \in A$ where $x \notin B$ or there is an $x \in B$ where $x \notin A$.
$A \neq \emptyset$	Prove $(\exists x)(x \in A)$.
$x \notin A \cup B$	Prove $(x \notin A) \wedge (x \notin B)$.
$x \notin A \cap B$	Prove $(x \notin A) \vee (x \notin B)$.
$x \notin A - B$	Prove $(x \notin A) \vee (x \in B)$.

To prove P by **contradiction**:

“Assume P is false”
then show that you arrive at a contradiction.

To prove $(\forall n \in \mathbf{N})(P(n))$ by **Mathematical Induction**, show the following:

- 1) $P(1)$ is true
- 2) For all $k \in \mathbf{N}$, if $P(k)$ is true then $P(k + 1)$ is true.