Math 199 Final Exam Review Sheet **Spring 2005**

For the final exam, it would help you to do the following things:

1. Learn the definitions you needed to know for the quizzes and for the midterm exam. Know the definitions precisely. You should know the statements in formal language, as well as interpretations in words. E.g., $A \subseteq B$ means $(\forall x)(x \in A \Rightarrow x \in B)$ and this can be read as "every element in A is also in B".

2. Do "blank-paper practice" for the problems on the problem sets and the midterm. Note: You should be able to do all the problems, including the hard ones. Avoid repeating a mistake you made on the problem set (this is important).

3. Understand these challenging concepts (plus the ones from the midterm):

a. The definitions of one-to-one and onto.
b. The image of a set under a function and how $x \in A$ relates to $y \in f(A)$.
c. The inverse image of a set under a function and how $y \in B$ relates to $x \in f^{-1}(B)$.
d. The difference between onto and $f(A)$.
e. The difference between $f^{-1}(B)$ (the set) and f^{-1} (the function).
f. The fact that $\sqrt{x}$ and x^2 are not inverses.
g. The difference between $[a]$ and a.

4. In addition to the proofs listed for the midterm, know the proofs of these key examples. You should not memorize them, but should remember the central idea(s) and reconstruct the proof from that memorized core.

a. If f and g are bijections then so is $g \circ f$ (PS3#5)
b. For $f\colon \mathbf{R} \to \mathbf{R}$ by $f(x) = x^2 + 1$, $f([-1, 2]) = [1, 5]$
c. $f(A \cap B)$ need not equal $f(A) \cap f(B)$ (PS4#4)
d. $f\colon X \to Y$ is onto iff $f(f^{-1}(B)) = B$ for all $B \subseteq Y$ (PS4#5)
e. $f\colon X \to Y$ is a bijection iff $f(X - A) = Y - f(A)$ for all $A \subseteq X$ (PS4#6)