

For the exam on Monday, it would help you to do the following things:

1. Learn the definitions you needed to know for the quiz. Know the definitions precisely. You should know the statements in formal language, as well as interpretations in words. E.g.,  $A \subseteq B$  means  $(\forall x)(x \in A \Rightarrow x \in B)$  and this can be read as “every element in  $A$  is also in  $B$ ”.
2. Do “blank-paper practice” for the problems on the problem sets that have been handed back, and for the homework problems. Note: You should be able to do *all* the problems, including the hard ones. Avoid repeating a mistake you made before.
3. Understand these challenging concepts:
  - a.  $\subseteq$  vs.  $\in$  vs.  $=$
  - b.  $A \subseteq B$  vs.  $A \cap B$
  - c.  $\mathcal{P}(A)$ , the power set of  $A$
  - d.  $(\forall x)(P(x))$  vs.  $\{x \mid P(x)\}$
  - e. “For all  $x, \dots$ ” vs. “There exists an  $x$  where  $\dots$ ”
  - f.  $(\forall x)(\exists y)(P(x, y))$  vs.  $(\exists y)(\forall x)(P(x, y))$
  - g.  $P \wedge Q$  vs.  $A \cap B$
  - h.  $P(x) \Rightarrow Q(x)$  vs.  $\{x \mid P(x) \text{ and } Q(x)\}$
4. Know the negations of the various types of propositions we’ve studied.
5. Know the contrapositive, converse, and inverse, and which ones are equivalent.
6. Know how to translate English into formal logic and *vice versa*.
7. Know how to do mathematical induction.
8. Know the proofs of these key examples. You should not memorize them, but should remember the central idea(s) and reconstruct the proof from that memorized core.
  - a.  $\emptyset \subseteq A$  for all  $A$
  - b.  $A \subseteq B$  iff  $\mathcal{P}(A) \subseteq \mathcal{P}(B)$
  - c.  $A \times B = \emptyset$  if, and only if,  $A = \emptyset$  or  $B = \emptyset$
  - d. For  $B \neq \emptyset$ , if  $A \times B \subseteq C \times B$  then  $A \subseteq C$
  - e.  $\sqrt{2}$  is irrational