

## Putnam Practice Problems

The following idea can be applied to solve each of the problems listed below. How? Hmmm...

**PIGEONHOLE PRINCIPLE** If  $kn + 1$  objects ( $k \geq 1$ ) are distributed among  $n$  boxes, one of the boxes will contain at least  $k + 1$  objects.

- (1) Given any five points on a sphere, show that some four of them must lie on a closed hemisphere.

.....

- (2) Consider any five points  $P_1, P_2, P_3, P_4, P_5$ , in the interior of a square of side length 1. Denote by  $d_{ij}$  the distance between the points  $P_i$  and  $P_j$ . Prove that at least one of the distances  $d_{ij}$  is less than  $\sqrt{2}/2$ .

.....

- (3) Let  $a_j, b_j, c_j$  be integers for  $1 \leq j \leq N$ . Assume for each  $j$ , at least one of  $a_j, b_j, c_j$  is odd. Show that there exists integers  $r, s, t$  such that  $ra_j + sb_j + tc_j$  is odd for at least  $4N/7$  values of  $j$ ,  $1 \leq j \leq N$ .

.....

- (4) Given a set of  $n + 1$  positive integers, none of which exceeds  $2n$ , show that at least one member of the set must divide another member of the set.

.....

- (5) Prove that there exist integers  $a, b, c$  not all zero and each of absolute value less than one million, such that

$$|a + b\sqrt{2} + c\sqrt{3}| < 10^{-11}.$$

.....

**Remark:** Each of these problems appeared on an actual Putnam Exam!