Special Geometric Structures

Program

Friday October 18

7:00-9:00 PM: Reception in Bailey Hall 204

Saturday October 19

8:00–9:00 AM: Registration and Coffee in Bailey Hall 204

9:00–9:30 AM: Gideon Maschler, Clark University: *Kähler metrics conformal to Ricci solitons.*

9:40–10:10 AM: Jeff Jauregui, Union College: *Conformal diffeomorphisms of generalized quasi-Einstein manifolds*

10:20–10:50 AM: Frank Morgan, Williams College: *Isoperimetric problems with densities.*

10:50–11:15 AM: Coffee in Bailey Hall 204

11:15–12:15 PM: No talk scheduled for our session but please check the program in parallel sessions.

12:15–2:10 PM: Lunch break

2:10–3:10 PM: Invited speaker: Claude LeBrun, Stony Brook University: *Curvature Functionals, Einstein Metrics, and the Geometry of 4-Manifolds.*

3:20–3:50 PM: Megan Kerr, Wellesley College: *New examples of non-symmetric Einstein solvmanifolds of negative Ricci curvature.*

4:00–4:30 PM: Hung Tran, Cornell University: *The Weyl Tensor of Gradient Ricci Solitons.*

4:30–5:00 PM: Coffee in Bailey Hall 204

5:00–6:00 PM: Invited speaker: William Wylie, Syracuse University: *Warped Product Einstein metrics and Ricci solitons.*

6:30 PM: Banquet in Old Chapel

Sunday October 20

8:30–9:00 AM: Coffee in Bailey Hall 204

9:00–9:30 AM: Tedi Draghici, Florida International University: *On almost complex 4-dimensional Lie algebras.*

9:40–10:10 AM: Janet Talvacchia, Swarthmore College: *Generalized Contact Structures and a Theorem of Morimoto.*

10:20–10:50 AM: Ralph Gomez, Swarthmore College: *On Certain Examples of 2-connected Quasiregular Sasaki-Einstein Manifolds in Dimension Seven.*

10:50–11:15 AM: Coffee in Bailey Hall 204

Abstracts

1. **Gideon Maschler, Clark University: Kähler metrics conformal to Ricci solitons.** Kahler metrics nontrivially conformal to Einstein metrics have been classified a few years ago on compact manifolds, and also locally in dimensions above four. For the corresponding problem of Kahler metrics conformal to gradient Ricci solitons, we present some partial results along related lines, giving information on such metrics in various cases characterized by the existence of a distinguished vector field. This work is in progress with Andrzej Derdzinski.

2. **Jeffrey Jauregui, Union College, Conformal diffeomorphisms of generalized quasi-Einstein manifolds.** In the 1950s, Yano and Nagano proved that the round sphere is the only complete Einstein manifold admitting a one-parameter family of conformal transformations. Since then, a number of results have been discovered regarding conformal diffeomorphisms of Einstein spaces. I will discuss recent work with William Wylie in which we extend these theorems to generalized quasi-Einstein (GQE) manifolds, a class of spaces that includes gradient Ricci solitons and static metrics. We also prove sharp characterizations of conformal transformations between shrinking and steady gradient Ricci solitons to other solitons. As with the classical results, we assume the conformal diffeomorphisms are non-homothetic, and we additionally assume the potential function is preserved up to a constant.

3. **Frank Morgan, Williams College: Isoperimetric problems with densities.** There has been a surge of interest in densities on Riemannian manifolds, especially since their role in Perelman's proof of the Poincaré Conjecture. In this context, the isoperimetric problem seeks to minimize weighted perimeter for prescribed weighted volume. We'll report on some recent progress following earlier preliminary work by undergraduates.

4. **Claude LeBrun, Stony Brook University: Curvature Functionals, Einstein Metrics, and the Geometry of 4-Manifolds.** An Einstein metric is by definition a Riemannian metric of constant Ricci curvature. This talk will begin by observing that the existence and uniqueness problems for Einstein metrics on smooth compact manifolds seem to behave quite differently in dimension 4 than in other dimensions.

   Many of the most powerful techniques currently available for the construction of Einstein metrics rely on results from Kahler geometry. We will see that this is not just a matter of happenstance. Indeed, the 4-dimensional Einstein metrics arising from Kahler geometry are objectively "better" than others, in that they are "less curved" than other metrics, as measured by various curvature norms. Specifically, we will explore the special role accorded to such metrics by the L^2 norm of the scalar curvature and by the L^2 norm of the Weyl curvature. Curiously, however, the roles of these two functionals turn out to be interchanged when one reverses the sign of Einstein constant!

5. **Megan Kerr, Wellesley College: New examples of non-symmetric Einstein solvmanifolds of negative Ricci curvature.** I will discuss two techniques for constructing new examples of non-symmetric Einstein solvmanifolds. I will first describe a method due to H. Tamaru (2011), in which he creates new (attached) Einstein solvmanifolds, using parabolic subalgebras of semisimple Lie algebras. These Einstein spaces are submanifolds, although not generally totally geodesic, of the solvmanifolds corresponding to noncompact symmetric spaces. I will then describe a method for extending his technique to create associated (but not isometric) Einstein solvmanifolds, using a method due to C. S. Gordon and M. Kerr (1999). Each technique is proved to preserve the (constant) Ricci curvature, even as the algebra and geometry are changed.

6. **Hung Tran, Cornell University: The Weyl Tensor of Gradient Ricci Solitons.** We discuss some new identities for the Weyl tensor of gradient Ricci solitons, specifically in dimension four. In particular, in the first part, we show a Bochner-Weitzenböck type formula for the norm of the self-dual Weyl tensor which has several applications including a gap theorem. In the second part, we are mostly concerned with the interaction of different components of curvature, the gradient and Hessian of the potential function. The Weyl tensor arises naturally in these investigations as it is the main distinction in higher dimensions. Applications here are rigidity results.

7. **William Wylie, Syracuse University: Warped Product Einstein metrics and Ricci solitons.** In this talk I'll discuss a characterization of Einstein metrics which are both homogeneous and admit a warped product structure. While metrics of these type are quite restrictive, I'll also show that one can always build such a metric from an algebraic Ricci soliton metric. The proof is motivated by some earlier results on homogeneous gradient Ricci solitons and follows from the study of an overdetermined linear system of equations on the manifold which generalizes a well known characterization of the sphere due to Obata. This is joint work with Peter Petersen of UCLA and Chenxu He of Oklahoma.

8. **Tedi Draghici, Florida International University: On almost complex 4-dimensional Lie algebras** Joint work with Hector Leon. We study the tame-compatible question of Donaldson for 4-dimensional almost complex Lie algebras and one other related problem regarding cohomology decomposition. Our observations can be considered as continuation and completion of earlier results of T.-J. Li and A. Tomassini.

9. **Janet Talvacchia, Swarthmore College: Generalized Contact Structures and a Theorem of Morimoto.** Generalized complex structures were introduced by Hitchin in 2005 and subsequently developed by his student, Gualtieri. The odd dimensional analogs, generalized contact structures, have been developed by Vaisman, Poon-Wade, and Sekiya. In this talk I'll give an overview of the notion of a generalized contact structure and discuss a recent result (joint with Ralph Gomez of Swarthmore College) concerning an analog of a theorem of Morimoto in this generalized context. The classical theorem of Morimoto discusses the existence of an almost complex structure on the cartesian product of manifolds that each admit almost contact structures. Our result concerns the existence of generalized complex structures on the cartesian product of spaces that each admit generalized contact structures.

10. **Ralph Gomez, Swarthmore College: On Certain Examples of 2-connected Quasiregular Sasaki-Einstein Manifolds in Dimension Seven.** In this talk, we discuss new examples of 2-connected seven dimensional Sasaki-Einstein manifolds realized as links of isolated hypersurface singularities for which some homology information can be obtained. This is done by using a valid case of Orlik's conjecture regarding homology groups of links. These examples pull from the extensive list due to J.Johnson and J.Kollár of Fano Kähler-Einstein orbifolds in weighted projective 4-space.