Union College Category Theory
Celebrating Bill Lawvere & 50 Years of Functorial Semantics

Program

Friday, October 18

7:00-9:00 PM: Reception in Bailey Hall 204

Saturday, October 19

8:00-9:00 Registration and Coffee in Bailey Hall 204

9:00-9:30 Dorette Pronk (Dalhouse University), *Orbifold function spaces*

9:40-10:10 John Kennison (Clark University), *Limit closures in rings and sheaf representations*

10:20-10:50 Steven Awodey (Carnegie Melon University), *Natural models of homotopy type theory*

10:50-11:15 Coffee in Bailey Hall 204

11:15-11:45 Peter Freyd (University of Pennsylvania), *Constructing the reals without Dedekind*

11:45-12:15 Michael Barr (McGill Unversity), *The limit closure of metric spaces in uniform spaces*

12:15-2:00 Lunch Break

2:00-2:30 Walter Tholen (York University), *Hilbert's Nullstellensatz and subdirect representation*

2:40-3:10 Rory Lucyshyn-Wright (University of Ottawa), *Riesz-Schwartz extensive quantities and vector-valued integration in closed categories*

3:20-3:50 Michael Makkai (McGill University), *The theory of abstract sets based on first-order logic with dependent types*

4:00-4:30 Ernie Manes (University of Massachusetts), *Ideas in functorial semantics, and topological dynamics*

4:30-5:00 Coffee in Bailey Hall 204

5:00-6:00 Bill Lawvere (University of Buffalo), *What are foundations of geometry and algebra?*

6:30 Banquet in Old Chapel

Sunday, October 20

8:30-9:00 Coffee in Bailey Hall 204

9:00-9:30 Robin Cockett (University of Calgary), *Tangent categories are locally Cartesian differential categories*

9:40-10:10 Geoff Crutwell (Mount Allison University), *Connections in tangent categories*

10:20-10:50 Dimitri Chikhladze (University of Coimbra), *Lax monads and generalized multicategory theory*

10:50-11:15 Coffee in Bailey Hall 204

11:15-12:15 Kathryn Hess (Ecole Polytechnique Fédérale de Lausanne) *The Boardman-Vogt tensor product of operadic bimodules*

12:15-1:45 Lunch Break

1:45-2:15 Pieter Hofstra (University of Ottawa), *Game theory, categorically*

2:20-2:50 Fred Linton (Wesleyan University), *How constructive is the old "group epimorphisms are onto" game?*

2:55-3:25 Noson Yanofsky (Brooklyn College), *Kolmogorov complexity of categories*

Union College Category Theory: Celebrating Bill Lawvere & 50 Years of Functorial Semantics

Key Note Lecture

What are Foundations of Geometry and Algebra?
F.W. Lawvere, University of Buffalo

From observation of, and participation in, the ongoing actual practice of Mathematics, Decisive Abstract General Relations (DAGRs) can be extracted; when they are made explicit, these DAGRs become a guide to further rational practice of mathematics. The worry that these DAGRs may turn out to be as numerous as the specific mathematical facts themselves is overcome by viewing the ensemble of DAGRs as a 'Foundation', expressed as a single algebraic system whose current description can be finitely-presented. The category of categories (as a cartesian closed category with an object of small discrete categories) aims to serve as such a Foundation. One basic DAGR is the contrast between space and quantity, and especially the relation between the two that is expressed by the role of spaces as domains of variation for intensively and extensively variable quantity; in that way, the foundational aspects of cohesive space and variable quantity inherently includes also the conceptual basis for analysis, both for functional analysis and for the transformation from continuous cohesion to combinatorial semi-discreteness via abstract homotopy theory. Function spaces embody a pervasive DAGR.

The year 1960 was a turning point. Kan, Isbell, Grothendieck and Yoneda had further developed the Eilenberg-Mac Lane Theory of Naturality. Their work implicitly pointed towards such a Foundation as a foreseeable goal. Although the work of those four great mathematicians was still unknown to me, I had independently traversed a sufficient fragment of a similar path to encourage me to become a student of Professor Eilenberg. As I slowly became aware of the importance of those earlier developments, I attempted to participate in the realization of a Foundation in the sense described above, first through concentration on the particular docrine known as Universal Algebra, making explicit the fibered category whose base consists of abstract generals (called theories) and whose fibers are concrete generals (known as algebraic categories). The term 'Functorial Semantics' simply refers to the fact that in such a fibered category, any interpretation $T' \to T$ of theories induces a map in the opposite direction between the two categories of concrete meanings; this is a direct generalization of the previously observed cases of linear algebra, where the abstract generals are rings and the fibers consist of modules, and of group theory where the dialectic between abstract groups and their actions had long been fundamental in practice. This kind of fibration is special, because the objects T in the base are themselves categories, as I had noticed after first rediscovering the notion of clone, but then rejecting the latter on the basis of the principle that, to compare two things, one must first make sure that they are in the same category; when the two are (a) a theory and (b) a background category in which it is to be interpreted, comparisons being models, the category of categories with products serves. Left adjoints to the re-interpretation functors between fibers exist in this particular doctrine of general concepts, unifying a large number of classical and new constructions of algebra. Isbell conjugacy can provide a first approximation to the general space vs quantity pseudo-duality, because recent developments (KIGY) had shown that also spaces themselves are determined by categories (of figures and incidence relations inside them).

My 1963 thesis clearly explains that presentations (having a signature consisting of names for generators and another signature consisting of names for equational axioms) constitute one important source of theories. This syntactical left adjoint directly generalizes the presentations known from elimination theory in linear algebra and from word problems in group theory. No one would confuse rings and groups themselves with their various syntactical presentations, but previous foundations of algebra had under-emphasized the existence of another important method for constructing examples, namely the Algebraic Structure functor. Being a left adjoint, it can be calculated as a colimit over finite graphs. Fundamental examples, like cohomology operations as studied by the heroes of the 50's, show that typically an abstract general (such as an isometry group) arises by naturality; to find a syntactical presentation for it may then be an important question. This extraction, by naturality from a particular family of cases, provides much finer invariants, and as a process bears a profound resemblance to the basic extraction of abstract generals from experience.

Contributed Talks

Natural Models of Homotopy Type Theory
Steven Awodey, Carnegie Mellon University

Homotopy type theory is an interpretation of constructive Martin-Löf type theory into abstract homotopy theory. It allows type theory to be used as a formal calculus for reasoning about homotopy theory, as well as more general mathematics such as can be formulated in category theory or set theory, under this new homotopical interpretation. Because constructive type theory has been implemented in computational proof assistants like Coq, it also facilitates the use of those tools in homotopy theory, category theory, set theory, and other fields of mathematics. This is the idea behind the new *Univalent Foundations Program*, which has recently been the object of quite intense investigation. One thing missing from homotopy type theory, however, has been a notion of *model* that is both faithful to the precise formalism of type theory and yet general and flexible enough to be a practical tool. Past attempts have relied either on highly structured categories corresponding closely to the syntax of type theory, such as the *categories with families* of Dybjer, which are, however, somewhat impractical to work with semantically, or more natural and flexible categorical models based on homotopical algebra, which however must be equipped with unnatural coherence conditions. In this talk, I will present a new approach which combines some of the good features of each of these two strategies. It is based on the observation that a category with families is the same thing as a representable natural transformation in the sense of Grothendieck. Ideas from Voevodsky and Lumsdaine-Warren are also used.

The limit closure of metric spaces in uniform spaces
Michael Barr, Mcgill University

Say that a net x_i in a uniform space is strongly Cauchy if for every pseudometric d, the net $d(x_i, x_j)$ is eventually 0. James Cooper conjectured and we (John Kennison, Bob Raphael, and I) proved that a separated uniform space is a limit of metric spaces iff every strongly Cauchy net converges.

Lax monads and generalized multicategory theory
Dimitri Chikhladze, University of Coimbra

Generalized multicategories, also called T-monoids are well known class of mathematical structures, which include diverse set of examples. In this talk we will construct a generalization of the adjunction between monoidal categories and multicategories, where the former are replaced by T -monoids. To do this we introduce lax monads in a tricategory, and establish their relationship with equipments, which are bicategory like structures appropriate for the generalized multicategory theory.

Tangent categories are locally Cartesian differential categories
Robin Cockett, University of Calgary

Recently Geoff Cruttwell and I, following Rosicky's original idea, introduced tangent categories as a setting for abstract differential geometry. We showed how this notion not only captured standard differential geometry settings, but also synthetic differential settings (SDG), and Cartesian differential setting. Recently, while considering differential bundles, we realized that there is an even deeper connection to Cartesian differential categories. Given any tangent category one can consider the differential bundles of that tangent category. These themselves form a tangent category which is a fibration over the original category. Each fibre, for very general reasons, inherits the structure of being a tangent category. Furthermore, one can then show that each fibre is actually a Cartesian differential category: hence the title.

Connections in tangent categories
Geoff Cruttwell, Mount Allison University

Following Robin Cockett's talk, I'll discuss how to define connections on vector bundles in the abstract setting of a tangent category. In ordinary differential geometry, connections are presented in many different ways. I'll show how our abstract definition of connection in a tangent category has a nice categorical description, but also encompasses many of the other standard definitions of connection from differential geometry.

Constructing the reals without Dedekind
Peter Freyd, University of Pennsylvania

The Boardman-Vogt tensor product of operadic bimodules
Kathryn Hess, Ecole Polytechnique Féacutedéacuterale de Lausanne

The Boardman-Vogt tensor product of operads endows the category of operads with a symmetric monoidal structure that codifies interchanging algebraic structures. In this talk I will explain how to lift the Boardman-Vogt tensor product to the category of composition bimodules over operads. I will also sketch two geometric applications of the lifted B-V tensor product, to building models for spaces of long links and for configuration spaces in product manifolds. This is joint work with Bill Dwyer.

Game theory, categorically
Pieter Hofstra, University of Ottawa

Categories of games qua models of logic or programming languages have been extensively studied over the past decades. However, the games appearing in such categories are necessarily limited in nature and are not nearly as general as the kinds of games considered in traditional game theory. We present a category-theoretic approach, based on locally cartesian closed categories and fibrations, to a general class of games, and explain how some important notions from classical game theory, such as game equivalence and solution concepts, can be interpreted in this setting.

Limit Closures in Rings and Sheaf Representations
John Kennison, Clark University

This talk is based on joint work with Mike Barr and Bob Raphael. We apply some general results about limit closures of full subcategories to certain subcategories of Rings. These limit closures are always reflective and, for rings, the reflection is often given as the global sections of a sheaf. Given a subcategory meeting our conditions, there is, for each semiprime (or nilpotent-free) ring, a corresponding sheaf over the spectrum whose stalks are domains in the limit closure. The topology on the spectrum is between the domain (or "co-Zariski") topology and the patch topology. Examples include the subcategories of all fields, of all domains, of all integrally closed domains, of all Bezout domains and of all GCD-domains (the last three examples have the same limit closure). Some open questions remain.

How constructive is the old "group epimorphisms are onto" game?
Fred Linton, Wesleyan University

At the recent Warsaw Samuel Eilenberg Centenary Conference, I tried to shed some new light on old arguments showing epimorphisms of groups are onto, viz., pointing out, as Andr Joyal had helped make me aware, that Sammy's take on that serves, in fact, to show that monomorphisms of groups are equalizers. Cf. http://fej.math.wes.tlvp.net/Eilenbg100-2013/talk.pdf .

The question having arisen, on MathOverFlow and other mathematical web-discussion venues, just how "constructive" that approach is, we offer here a tentative partial answer: it's perfectly constructive for inclusions of subgroups H of G that are complemented in G.

Indeed, that complementedness is just what it takes for the singleton $\{H\} : 1 \to G/H$, consisting solely of the coset H itself, to be a complemented subobject of G/H, which is exactly what is needed to permit construction of the permutation of $G/H+1$ that Sammy's argument exploits, exchanging the summand $\{H\}$ of G/H with the summand 1 of $G/H+1$, yet leaving the complement of $\{H\}$ in G/H alone.

Our strategy is to notice that, for a coproduct $X = A+B$, it's easy to extend a permutation of A and/or of B to a compatible permutation of X. Alas, there are toposes with object X, subobject A of X, and permutation p of A, for which there is no permutation of X extending p, but while that destroys the effectiveness of the proof strategy, it does not provide a counterexample to the "epis are onto" statement itself.

Riesz-Schwartz extensive quantities and vector-valued integration in closed categories
Rory Lucyshyn-Wright, University of Ottawa

We develop aspects of functional analysis in an abstract axiomatic setting, through monoidal and enriched category theory. We work in a given closed category, whose objects we call *spaces*, and we study R-module objects therein (or algebras of a commutative monad), which we call *linear spaces*. Building on ideas of Lawvere and Kock, we study functionals on the space of scalar-valued maps, including compactly-supported Radon measures and Schwartz distributions. We develop an abstract theory of vector-valued integration with respect to these scalar functionals and their relatives. We study three axiomatic approaches to vector integration, including an abstract Pettis-type integral, showing that all are encompassed by an axiomatization via Eilenberg-Moore algebras and that all coincide in suitable contexts. We study the relation of this vector integration to relative notions of completeness in linear spaces. One such notion of completeness, defined via enriched orthogonality, determines a symmetric monoidal closed reflective subcategory consisting of exactly those separated linear spaces that support the vector integral. We prove Fubini-type theorems for the vector integral. Further, we develop aspects of several supporting topics in category theory, including enriched orthogonality and factorization systems, enriched associated idempotent monads and adjoint factorization, symmetric monoidal adjunctions and commutative monads, and enriched commutative algebraic theories.

The theory of abstract sets based on first-order logic with dependent types,
Michael Makkai, McGill University

On my website (http://www.math.mcgill.ca/makkai/), the last item in the list “Papers” you find a recent paper of mine with the same title, referred to as “the paper” below. Although it has not been published (or even submitted for publication), it is a complete paper in all reasonable aspects. My talk will consist of mathematical excerpts of the, on the whole rather philosophical, paper. I will discuss a passage from F. W Lawvere’s famous 1976 paper “Variable quantities and variable structures in topoi.”. A crucial sentence of said passage amounts to a call to produce a formal language for abstract sets in which all statements on a single variable (abstract) set are invariant under equipollence of sets. I produce such a language (a particular case of FOLDS: First Order Logic with Dependent Sorts), and state and prove “Lawvere’s imperative”, the required invariance property for the language, as well as a natural generalization of it that I call “Benaceraff’s imperative”. The paper also attempts to demonstrate the expressive power of the language in a way that seem to be new. The paper has a detailed historical discussion from which it will be seen that many ingredients of the paper are not new, having precursors in works of other authors as well as in my own work. However, I view the mathematical formulations and proofs of the “imperatives” to be new to the paper.

Bill Lawvere’s ideas in functorial semantics, and topological dynamics
Ernie Manes, University of Massachusetts

In the qualitative theory of dynamical systems,a perturbed periodic point (e.g. Halley’s Comet) is “almost periodic”. In the 1960s, Robert Ellis associated to a compact Hausdorff group action X a monoid $E(X)$ of “infinite times” which act on X. Then x in X is almost periodic if there exists t in $E(X)$ with $tx = x$.

The monoid structure of $E(X)$ is explained by observing that it is the free algebra on one generator in the variety V generated by X. In particular, if Y is in V, a component of the resulting theory map establishes a surjective monoid homomorphism $E(X) \to E(Y)$ inducing, in turn, the usual left and right adjoints.

The traditional universe of compact Hausdorff spaces abandons the various countability conditions that find use in thinking about dynamical systems. For example, a large power of 2 need not be countably tight. But the category of countably tight spaces does have products. There are algebraic categories of dynamical systems which contain all compact metric systems and with all state spaces countably tight. While the variety generated by X here requires infinitary operations, the full subcategory of singly generated algebras is isomorphic to the category of one-orbit $E(X)$-actions and this is a finitary description. This is relevant because, here, x is almost periodic if and only if it generates a minimal subalgebra. This generalizes a famous result of the elder Birkhoff that dates to 1912.

Orbifold Function Spaces
Dorette Pronk

I will discuss the orbispace structure on a mapping space of orbifolds in terms of proper etale groupoids. In the process we will see how the bicategory of fractions can be viewed as a pseudo colimit of categories of fractions. I will also present some concrete examples and if there is time I will show how the inertia orbifold of an orbifold can be viewed as a mapping space into that orbifold.

Hilbert's Nullstellensatz and subdirect representation
Walter Tholen, York University

David Hilbert's solvability criterion for polynomial systems in n variables from the 1890s was linked by Emmy Noether in the 1920s to the decomposition of ideals in commutative rings, which in turn led Garret Birkhoff in the 1940s to his subdirect representation theorem for general algebras. The Hilbert-Noether-Birkhoff linkage was brought to light in the late 1990s in talks by Bill Lawvere. The aim of this article is to analyze this linkage in the most elementary terms and then, based on our work of the 1980s, to present a general categorical framework for Birkhoff's theorem.

Kolmogorov Complexity of Categories
Noson Yanofsky, Brooklyn College

Kolmogorov complexity theory is used to tell what the algorithmic informational content of a string is. It is defined as the length of the shortest program that describes the string. We present a programming language that can be used to describe categories, functors, and natural transformations. With this in hand, we define the informational content of these categorical structures as the shortest program that describes such structures. Some basic consequences of our definition are presented including the fact that equivalent categories have equal Kolmogorov complexity. It is also shown that our definition is a generalization of Kolmogorov complexity theory of strings. We also discuss what can and cannot be described by our programming language.