MATH AWARENESS MONTH 2000: AN INTERACTIVE EXPERIENCE

THOMAS F. BANCHOFF AND DAVIDE P. CERVONE

1. Introduction

Mathematics Awareness Week was founded in 1986 by the Joint Policy Board for Mathematics (JPBM) as a public relations effort to increase appreciation for the power, scope and beauty of mathematics. In 1999, the campaign was extended to Mathematics Awareness Month, held annually in April.

Each year, Mathematics Awareness Month is the responsibility of one of the three organizations that make up the JPBM: the American Mathematical Society (AMS), the Mathematical Association of America (MAA), and the Society of Industrial and Applied Mathematics (SIAM). For MAM2000, that role fell to the MAA.

Math Awareness Month always has a theme approved by the JPBM as a whole. Themes in the past have included "Mathematics and Symmetry", "Mathematics and Finance", and "Mathematics and the Internet". In 1999 and 2000, the first-named author was President of the MAA, and he suggested "Math Spans All Dimensions" as the theme for 2000. This concept was approved in May 1999 and the preparations began.

The centerpiece for each MAM campaign is a poster, designed to be displayed in classrooms and on bulletin boards at colleges and universities, to draw attention to the theme and to lead students and teachers to delve further into the topic by reading ancillary materials, usually in the form of short essays on sheets distributed along with the poster. For MAM2000, we decided to design and implement an electronic poster that would put the additional materials directly at the hands of the viewer, and appeal to audiences at different levels.

The key metaphor for the electronic poster is that of an Advent Calendar. Such a calendar usually is made from cardboard, containing small rectangles numbered for the days of December leading up to Christmas. The rectangles are flaps, and each day one flap is opened to reveal some toy or other present, a promise of things to come.

Our idea was to create an electronic analogue of such a calendar, with small rectangular icons or pictures that could be selected to open up more and

more information and further links. Other images and words on the poster would be "live" as well, inviting exploration into other aspects of the theme. From the beginning, we intended to develop a central geometric object that would organize the theme, accompanied by pictures of individuals associated with "Mathematics Across All Dimensions".

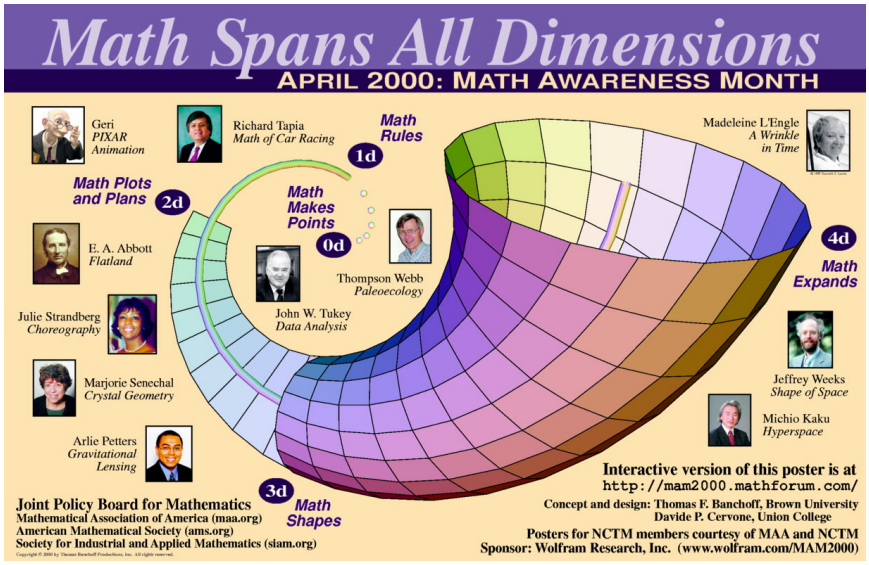

FIGURE 1. The print version of the MAM2000 poster includes a reference to the electronic version [11], where each picture and caption is a link to more information.

From the outset, the two authors collaborated by phone and email to bring this idea to fruition. The production of the MAM2000 poster involved a number of challenges, both technical and organizational, and the story of its development and implementation is the subject of this paper. In the sections below, we treat both the technical aspects and the ones dealing with the participants.

2. BEGINNINGS

On one afternoon in the fall of 1999, when the first-named author was visiting the second at Union College for a day, many of the basic design decisions fell into place. First, we settled on the central organizing image, a "cornucopia" formed by expanding circles perpendicular to an exponential spiral, somehow symbolizing the progression from zero to four dimensions. Our initial sketches suggested that the poster should be horizontal rather than the usual vertical arrangement. The practical size for a bulletin board poster was set at $11''$ by $17''$, so ours was $17''$ by $11''$. For mailing purposes, the poster would be folded so it could fit in a standard envelope. We wanted to

avoid having the crease intersect the central cone in an inconvenient way, and that determined the image placement in our design. Finally, we sketched out the arrangement of pictures around the cone, as well as the placement of acknowledgements and sponsorships. Remarkably enough, that initial basic sketch survived all the refinements and changes that we made over the next six months.

The selection of spokespersons whose picture would appear on the poster raised some challenges. We wanted to be unapologetically representative in our choices, with respect to fields and applications, and also with respect to age, gender, and race. We had a good pool to choose from in all categories.

Paul Tukey is the father of interactive multidimensional analysis. He had participated actively in our 1984 interdisciplinary symposium, "From 'Flatland' to Hypergraphics". He was happy to be included, but he felt that he would not be able to contribute much that was new. We already had material from him for the earlier symposium and we were counting on help from other researchers in exploratory data analysis. That set up the first picture in the 0-dimensional section, subtitled "Math Makes Points".

Thompson Webb III is a colleague of the first author in the Geology Department at Brown University who has participated in a variety of interdisciplinary activities related to space and time. He contributed a major section on core-sample research in paleoecology to the author's volume *Beyond the Third Dimension* [2]. This section maps the prairie-forest boundary within the midwest United States over a ten-thousand-year period by analyzing fossilized pollen concentrations in yearly depositions in lake beds. Since that chapter and its associated materials were already prepared, this was an obvious choice for the beginning of the 1-dimensional section, tentatively titled "Math Rules".

Geri, the star of the Oscar-winning short animated film "Geri's Game" [9], also making a guest appearance in the feature film "Toy Story II" [10], is the only fictional person pictured on the electronic poster. Tony de Rose and Ed Catmull of PIXAR Studios met the author at a conference on computer-aided geometric design in November 1999 and both agreed to participate in the project. They arranged to have Geri featured on the poster, with links to articles on the use of mathematics in computer animation, in particular the frontier research topic of subdivision surfaces. It was a definite plus that some of the images in the article about subdivision surfaces related directly to the Ph. D. thesis problems of both authors. This was the first topic in the 2-dimensional section, tentatively titled "Math Plots and Plans".

Edwin Abbott Abbott is the only deceased person who was chosen to appear in the gallery. As the author of *Flatland* [1], he is the founder of the dimensional analogy for understanding and communicating between dimensions. In addition, just the previous year we had assembled materials for a

Brown University Library exhibit, "*Flatland*, a Millennial Book" in conjunction with the MAA 1999 MathFest held in Providence, RI. We already were committed to producing interactive Internet-based materials connected with the publication history of *Flatland* and the life and times of its remarkable author. We previously had accumulated a great number of links about this topic, in particular electronic records of final projects by students in the first author's frequent courses on "The Fourth Dimension".

Julie Strandberg, chief choreographer at Brown University, is another long-time collaborator of the first author. An account of her dance composition "Dimensions", a retelling of *Flatland* themes, already appeared in a section of *Beyond the Third Dimension* [2]. There were other links available to sites on modern dance, and symbolic notation for classical and modern dances in a variety of cultures.

Marjorie Senechal is a world leader in mathematical crystallography and an organizer of a major conference on polyhedra. She agreed to write about her recent work and to provide photographs of a collaboration with the mathematician and sculptor Helaman Ferguson. This last topic, with its connections both to planar tilings and to solid crystals, is the bridge to the 3-dimensional part of the poster, with the title "Math Shapes".

Arlie Petters, a mathematician specializing in astronomy, specifically in the theory of gravitational lensing, has interacted with the first author for a number of years, and he had produced excellent visual presentations of his work. Despite a busy schedule, he was quite willing to participate and to help us find links to other related sites.

Michio Kaku, author of the recent volume *Hyperspace* [6] and numerous other writings on modern mathematical physics, had contacted the first author concerning background images for a television interview. He was also happy to participate and suggest a number of links to other sites, although he did not have time to develop new materials for the poster.

Jeff Weeks could be counted on to contribute at all levels since he already had produced impressive and accessible materials on topology and geometry, especially in three and four dimensions. He had just been named a MacArthur Fellow, and he was willing not only to appear on the poster, but also to help with the writing and the production of the interactive applets for viewing 4-dimensional phenomena. His section fit in with 4-dimensional geometry, with the title "Math Expands".

Madeleine L'Engle is a colleague with whom the first author has worked for years, primarily in connection with her extremely well-known prize-winning book, *A Wrinkle in Time* [7]. For many students, this fantasy was their first introduction to the fourth and higher dimensions and to the 4-dimensional

FIGURE 2. The page about Jeff Weeks describes him and his work, and includes links to software that he produced, articles in which he is featured, his book *The Shape of Space* [8], and movies related to that book.

cube, also known as the *tesseract*. One of her granddaughters, who is currently helping her with her projects, was very encouraging and helpful in arranging for her participation in the poster. There were many links available for this site, including a final project by a group of students in a course the first author taught at Yale as a visiting professor in the fall of 1998.

All of the contacts with those who appeared on the poster were made in the mid-fall of 1999. Aside from their connection with dimensionality in their work, the participants have a number of other things in common. First, the first author had worked with all of them over the years and had maintained good contact with them. This was essential considering the short amount of time we had for assembling the list of participants. Second, most reported that they had little or no time to work on producing anything new for the project, so it was clear that we ourselves would be doing most of the writing and producing demonstrations.

Only one person who had been contacted declined to participate, since the work of that person over the past ten years had veered away from topics connected with the theme of the poster. That meant that there was one slot left to be filled. Several colleagues who were consulted suggested the same person for a position in the 1-dimensional part of the poster, namely an applied mathematics and engineering professor who was also known as a champion drag racer! Richard Tapia was recommended as well for his exemplary work for access to higher education for Latino and Native American students, and he saw this poster as fitting in with his work on several levels. His assistant provided us with excellent materials for the poster.

Our selection process for the persons to appear on the poster was now complete, with a few months still to go.

The next task in the project was to make arrangements with the MathForum [12], which had provided the web server for earlier Math Awareness

Month (MAM) materials. This became a straightforward matter, without complications, due to the excellent work of Gene Klotz, with whom we had worked on other projects over the years. Although our concept of an interactive electronic poster was orders of magnitude more involved than previous MAM efforts, we received full cooperation from the MathForum leaders and technicians in setting up MAM2000.

In recent years, there has been corporate sponsorship for the MAM poster, and this year negotiations were already underway with Wolfram Research, Inc. The basic concept that we proposed found favor, and we worked with Paul Wellin for the next several months on various aspects of the project. One that took more time than we expected had to do with the central image. We had produced our prototype using the `CenterStage` program developed by the second-named author, but it was considered appropriate to render the central figure using Mathematica. One of the designers at Wolfram also proposed a central image along with a color scheme and background design. These were brought, along with the sketch by the authors, for consideration by the sponsoring Joint Policy Board for Mathematics. That body accepted the color and background proposed by the designers from Wolfram, but they preferred the simpler central figure of the authors as fitting in better with the theme of proceeding through various dimensions.

3. Technical Development

Once the nature of the central figure was determined, we proceeded to work on several levels simultaneously. We finalized the mathematical design of the exponential cone and worked on the layout and organization of the poster itself. We began accumulating photos of the participants, and we produced drafts of the pages describing the people and their work. The majority of the pages were written by the first-named author, assisted at various times by Jeff Weeks and Marjorie Senechal. As the pages took shape, we began the search for appropriate links to add to these pages. Several students in the first author's class helped to identify these links, and Frank Farris not only located materials but also helped to design a page of resources on two-dimensional patterns. The second author handled nearly all of the technical aspects of the site design and implementation, organizing the layout of all the pages, and producing the movie clips that are part of the site. Most of the interactive applets were produced by students at Brown University, while one applet for manipulating the hypercube was developed by Jeff Weeks specifically for this project.

To indicate the progression of dimensions along the central cornucopia image, we employed a series of different forms along the central exponential curve. We represented zero dimensions by a string of points along the curve, becoming a thickened curve to depict one dimension. For two dimensions,

we displayed the curve together with a segment along the principal normal vector (the segment in the plane of the velocity and acceleration vectors of the curve, perpendicular to the velocity vector). The segment was centered on the curve and we chose to have it grow linearly rather than exponentially, for artistic reasons. To represent three dimensions, we used circles centered on the curve in planes perpendicular to the curve, with radius continuing to grow linearly. We used an exponential spiral with a carefully chosen exponent, and a linear function determining the radii of circles so that this conical surface would open just the right amount for the space available on the poster. Although it is not immediately apparent, the exponential spiral is not a curve in the plane but rather a space curve, where the third coordinate is a linear function of the parameter that defines the curve. This helps create the effect that the cone is "opening up" at the end, so that we can see into it. The viewer is encouraged to imagine how the cornucopia would continue into the fourth dimension and higher!

In the final electronic version of the poster, the formulas we used are included in one of the links that appear when any point on the central figure is selected. In all of these pages, we attempted to describe the objects in ordinary language initially, but provided the more sophisticated reader with the opportunity to obtain progressively more technical discussions of the material. We reproduce the key paragraphs of that analysis here [13]:

> To describe our central image in technical language, we have to define a central curve $X(t)$ over a parameter t. The curve we chose is an arithmetic spiral lifted onto a cone, with equation
>
> $$X(t) = (t\cos(t), t\sin(t), \pi t).$$
>
> The radius function $r(t)$ that describes the expanding cone around this spiral is given by $r(t) = 0.8e^{0.6t}$.
>
> As t goes from 0 to $\pi/2$, we show a number of points on the curve. From $\pi/2$ to π, we show the curve itself, actually represented by a thin tube.
>
> From π to $3\pi/2$, we show a "normal strip" of the form $X(t)+rP(t)$ where r goes from $-r(t)$ to $r(t)$. Here $P(t)$ corresponds to the principal normal, a unit vector perpendicular to $T(t)$, the unit tangent vector of the curve, and lying in the plane determined by the velocity and the acceleration of the curve.
>
> From $3\pi/2$ to 2π, the figure is an expanding cone, or "cornucopia". The equation of the circular slice for each t is
>
> $$X(t) + r(t)(\cos(u)P(t) + \sin(u)B(t))$$

where $B(t)$ is the unit binormal vector perpendicular to $T(t)$ and $P(t)$, and where u is a parameter that goes from 0 to 2π.

Thus the image grows not only in size, but also in dimension.

As we finalized our design for the central cone, we began to put the images of the participants in place on the poster. This was an evolving process. New pictures were added as we found them and obtained permission to include them. We produced a preliminary poster using low-resolution images from the web as a sample to show to the JPBM at their December 1999 meeting. After the design was approved, the MAA selected the poster as the cover art for the first full-color issue of their news publication *Focus* [4] in March of 2000. The choice was made just a few days before the final artwork was needed by the publishers, so we had to move quickly to get everything together in time for their deadline. In particular, we did not at this point have any of the high-resolution versions of the images of the participants. Remarkably, we were able to obtain photos or scans from everyone within just two days of our request! This allowed us to put the final print version together in time for use by *Focus*. Their original design called for the poster to be used for both the front and back covers (the 17×11 image would be folded in half to form both covers); but the large end of the cone made for a less successful image than we had anticipated, and the editors decided to put the entire poster on the front cover. The reduced size would have made the images of the participants too small to reproduce well, so we chose to print only the central cone together with the wording from the poster. The result was extremely successful, and generated numerous requests from MAA chapters and other organizations to use the image on their own publications.

Final touches were made to the poster during the following week, and by mid February the main poster was ready to go to print. An initial run of 4,000 posters was produced and mailed to mathematics departments in colleges and universities around the country. A second run of 120,000 was sponsored by the MAA and the National Council for Teachers of Mathematics (NCTM) and mailed out to all NCTM members during the first week of April.

4. Putting it All Together

Once the print version of the poster was complete and at press, we were left with a month and a half to put the electronic version together before the April 1st opening date. In reality, we needed to be on line much earlier. All parties concerned were anxious to have a version available before the beginning of April, especially since the site had been announced in the March issue of *Focus* [4]. The need to make as much of the site "live" as possible, while still allowing for rapid change and prototyping kept both authors very

busy (while they were performing normal teaching duties in addition to developing this site).

Our vision for the design of the interactive web site was based on several themes. First, we wanted a multi-level approach, through which any reader could have access to general information about the topics of interest, and could continue to follow links that would bring them more deeply into the subject. Thus, for example, someone interested in the main cornucopia image could click on that to get a general description of the object, and then could follow additional links to the details of the equations involved, or to movies and interactive versions of the cone. A person interested in dance could see photographs of a dance sequence, or sites devoted to dance notation or ballet.

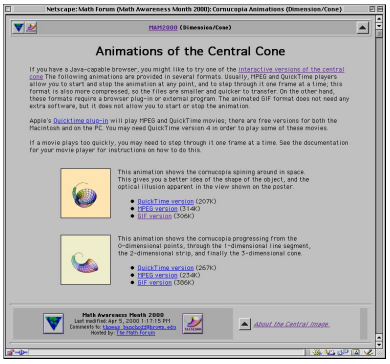

FIGURE 3. The page of movies for the central cornucopia provides these movies in several formats, and describes what player is needed for each.

Second, since the site was to be accessed by students in classrooms around the country, we wanted the pages to load quickly and be viewable by a wide variety of browsers. This meant that we wanted to use a minimum of "cutting-edge" HTML features, to make the material accessible to the widest possible audience, while still forming an attractive site. Aside from the opening page with its large image of the poster, most pages used only small images that download relatively quickly. Furthermore, large or time-consuming items, such as movies and Java applets, do not appear unless the reader specifically requests them. For example, the movies of the central cone appear on a separate page specifically for these movies, not on the one linked to the cone from the main poster. This approach also made it possible to provide the movies in several formats, so the reader can select the one that is most appropriate for his or her connection and software configuration.

Third, we wanted to be sure that there was a consistent "look and feel" to the site. In particular, we wanted all the pages to have connections back to the main poster page and the MathForum. Since many users would be arriving at the site by following links from search engines, it was crucial to give them a clear idea of where they had arrived, and how the page they found fit into the structure of the MAM2000 site. We wanted to avoid the

frustration of landing on a page that does not provide any connections to the rest of the material at that site.

We paid particular attention to the navigation buttons for our site. The header was designed to make it easy to move from page to page or back to the home page. These buttons were placed at the top of the page, so that they could be used without the need to scroll the page, and they are small and unlabeled so they don't take up much room. At the bottom we placed similar buttons, this time including the titles of the pages to which they refer. They are at the bottom, so that someone who has read through the document would not need to scroll back up to the top to get at them. We included the titles at the bottom of the page as well, to give more information about possible continuations.

Fourth, we wanted to use the structure of the site to mirror the structure of the material being discussed. The main theme, that of the sequence of dimensions, appears in several places within the poster. We have already seen how the cornucopia displays this progression. Originally, we imagined that there might be a sequence of images on the poster itself, going from a point to a segment to a square to a cube and then to a hypercube, perhaps sitting inside the cornucopia. This idea was abandoned since it was felt that the design was already quite busy, and it became clear that it was possible to develop this and other notions in a series of small essays directly accessible from the labels "0d", "1d", and so forth. Each of these short essays gave an overview of the images associated primarily with that dimension, and introduced some further analogies. In each case there was an attempt to distinguish between the "intrinsic" dimension of an object and the "extrinsic" dimension of the "ambient space" which contained it. The intrinsic dimension of an object is the number of parameters necessary to locate each point along the object, while the "extrinsic" dimension of an object in space is determined by the number of coordinates in the space where the object is situated.

In addition to being linked to the dimension labels on the main poster, each of these essays was linked to the next. In a similar way, we linked sequentially the pages about the various people featured on the poster, so readers could continue to the next person without having to return each time to the poster itself.

One more sequence of this sort was developed within the dimensional essays connected with the "dimensional buttons" labeled "0d" through "4d" on the poster. These include a sequence of interactive demonstrations on rotations of cubes and hypercubes. In dimension two, a square with four vertices $(\pm 1, \pm 1)$ can be rotated while a matrix shows the coordinates of the images of the first and the second basis vectors. A similar demonstration in dimension three represents a cube with its eight corners at $(\pm 1, \pm 1, \pm 1)$,

and allows the viewer to rotate the cube interactively and see the rotation matrix involved in producing that view. Finally, a demonstration for the hypercube extends these ideas into the fourth dimension.

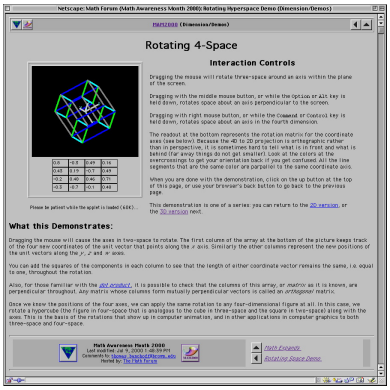

FIGURE 4. This Java applet demonstrates rotations in four dimensions; the matrix at the bottom shows the current rotation. The instructions for the user are to the right of the applet, and a mathematical discussion follows. This page is linked to the applets for the other dimensions, as well as to the essay on the fourth dimension.

Since these demonstrations are Java applets, they appear on pages of their own, complete with instructions on how to use them, and are linked from the essay on the appropriate dimension; but they also are linked to each other so that a user can go easily from experimenting with one dimension to investigating the analogous phenomena in the next.

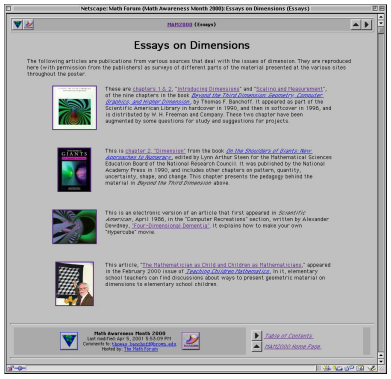

FIGURE 5. Several chapters from publications about dimensions are reproduced here, for viewers who are interested in more extensive details.

Another important portion of this site is not linked directly to the poster itself, but appears in the list of links below it. This is the collection of "essays on dimension". Two chapters from the first author's book, *Beyond the third dimension* [2], are reproduced here. We were able to obtain an electronic copy of the text from the publisher (with their permission), and since the second-named author had produced the line art for the book, we were able to regenerate these images for the web fairly readily. Several pictures required more work, however; these were originally produced using programs that were no longer available to us, and so they had to be reproduced in more modern software. For example, the three images showing the swallowtail catastrophe surface are not the original ones from the book, but were recreated specifically for the MAM2000 site.

Similarly, a chapter from *On the Shoulders of Giants* [3], published by the Mathematical Sciences Education Board of the National Research Council, also is included in this collection. Again, the text and graphics had been produced by the authors, so we were able to create the pages and images for the site quickly. The last essay, A. K. Dewdney's "Four-Dimensional Dementia" [5], originally appearing in *Scientific American* in 1986, required us to scan the artwork, producing somewhat lower-quality results.

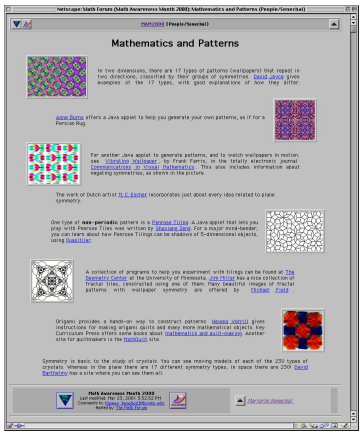

FIGURE 6. This list, available from Marjorie Senechal's page, provided links to a number of internet sites about two-dimensional patterns. There are lists like this on several other pages, as well.

These essays provide more extensive background for those interested in pursuing the idea of dimension further. Other resources are included in the form of lists of videotapes about dimensions, titles of books about dimensions, and links to web sites related to dimension.

5. The Final Touches

As April approached, the web site began to receive more attention from outside readers. To make it easier for other sites to link to the MAM2000 poster, we provided MAM2000 icons in several sizes, together with HTML code that could be copied directly into a web page to produce a link to the MAM site. These were used by dozens of math departments in conjunction with their Math Awareness Month activities, and the images also appeared on other math-related websites throughout the country.

FIGURE 7. This image was made available for web administrators to include on their mathematics pages as a link to the MAM2000 site. We even provided the HTML code to make the link.

One of the original issues we faced when we began to develop the MAM2000 site was how large to make the main image. We wanted the poster to be able to fit on the small-sized screens at schools with fairly old equipment

(e.g., screens that are 640×480 pixels). On the other hand, we wanted to provide a larger image for those with access to larger screens. We ended up providing three separate solutions to this problem.

First of all, we produced the main image in three different sizes, 612×396, 765×495 and 918×594. The viewer could switch between these sizes by clicking on one of the "poster size" buttons to the right of the image of the poster. Thus if the user wanted to use a larger (or smaller) version, it was possible to select the most appropriate one for the equipment at hand. The pages were set up so that going back to the home page would bring the reader back to the size last selected.

Second, we arranged that when the MAM2000 home page was first loaded, it would run a JavaScript program that would attempt to determine the size of the browser's window and select the largest of the three sizes that would fit comfortably on screen. This works properly for Netscape and Internet Explorer, provided JavaScript is enabled; the correct page would be selected automatically without the user even being aware that a choice was being made. For those without a browser that processes JavaScript (or who have it turned off), the 612×396 version would be used by default.

We gave visitors with a small screen, who would like to see an enlarged version of the poster, a third mechanism to make this possible. To the right of the poster are a set of small icons representing four quadrants of the poster. Clicking on a quadrant enlarges that region of the poster by a factor of two in each direction. All the links to portions of the poster that appear on a magnified section still work, and the viewer can move directly from one quadrant to another without returning to the full poster.

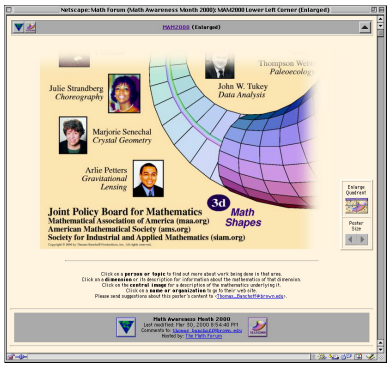

FIGURE 8. Visitors to the MAM2000 site can choose to enlarge one quadrant of the poster. The links from the poster are all still available, and any of the other three quadrants can be selected via the buttons at the right.

While it is somewhat risky to make web pages too "smart", this scheme seems to have worked out well. The browser makes an educated guess about what size and view the user wants, and then that size can be modified as desired. Those with older browsers (which do not process JavaScript) can still have access to the larger sizes without even knowing they are missing anything.

With these final changes in the last week of March, we were ready to "go live" on the 1st of April, as planned.

6. The Site in Use

The opening of the MAM2000 site on April 1st was a success, and we received numerous complimentary mail messages that month. We were eager to see the usage statistics collected from the log data from the web server, and were kindly given the raw data by the staff of the MathForum. It is not easy to interpret such information, however. Most people simply discuss "web hits", which is the total number of files shipped by the server, but this is a poor measure of the usage of the site for a variety of reasons. First, each image on a page is shipped separately, so a page with many pictures will generate many hits, artificially inflating the hit count. Similarly, a single Java applet may cause the downloading of numerous `.class` files. Second, many web-based search engines send out "robots" to read pages on the web so that they can be indexed and included in the search engine's database. Some of these robots are quite aggressive, reading the entire web site on a regular basis. Such hits should not count as "real" hits, since they don't correspond to people actually reading the web site. Depending on how well linked to search engines a site is, this can be a significant portion of the traffic from the web site. Finally, because browsers and proxy servers can cache commonly loaded pages, the log data may not include all the times that some users actually *do* view the pages.

For all these reasons (and more), any web log data should be viewed with a healthy dose of skepticism, and used only as a rough idea of the actual usage. Filtering the data to attempt to remove the effects can make a big difference in the results. While it is fairly easy to remove images from the hit count, there is no reliable way to distinguish "real" hits from those made by automated robots, and no way at all to count the hits that are serviced by caches and proxy servers.

Rather than a pure hit count, a more interesting statistic would be a "session count" indicating how many people viewed the site over a given time period. Such information is extremely difficult to come by (without special server software). The information in the log says that a file was shipped to a particular machine at a particular time, but one can only make educated guesses about when several such events fit together into one "session" by a single user at that machine. One approach is to use the times that the files were sent to a given machine, together with information (not always available) about the previous page being viewed by the user to try to piece together the sessions. This is somewhat inaccurate, but does give a reasonable sense of the number and length of sessions for the web site.

In our case, the data obtained from the MathForum included hits to other parts of their web site that needed to be removed, and seemed to be missing some of the hits to the MAM2000 pages. (Log data indicated that a link was made from another page of the MAM2000 site, but no record was included of that page being served to the indicated machine.) The following data gives the best information we have about the usage of the MAM2000 pages.

In the month prior to the official opening of the site, we averaged about 800 (non-robot) sessions or about 2,100 (real) pages per week. Not surprisingly, in the first week of April we reached our peak usage of about 3,900 sessions (16,000 files) per week. The following weeks showed a decrease to 3,100, 2,800, and 2,300 sessions per week, and after April, there was a significant drop off (as expected), to about 500 sessions a week during the following month. The total usage in April was approximately 12,200 sessions (52,700 files), or about 400 sessions a day of around 4.3 pages each. The usage by robots climbed steadily in the month of May, until it accounted for about a third of the total usage of the site, with more than 1,500 files being shipped to over 30 different robots per week.

7. Conclusion

Going beyond the traditional poster format to an electronic document involved a number of challenges to be met over a short period of time. Although the MAM2000 poster is still accessible [11], the number of visitors is down to a very small number. The ephemeral character of the project becomes clear as current visitors encounter more and more links to external sites that simply do not work. We tried hard to keep things operating during the actual Math Awareness Month, and for the rest of the year 2000, but now it seems inevitable that the outside links will decay faster and faster. Thus it is hard to archive this kind of electronic document. To achieve any permanence, we would have to collect all the files to which we linked, a difficult task under any circumstances for a project of this size. It would have been impossible to secure the required permissions to accumulate these files within the time frame for the project. We hope sometime in the future to explore creating a permanent, self-contained version of such a two-dimensional window on "Math Across All Dimensions", based on our experience with Math Awareness Month April 2000.

References

[1] A. E. Abbott, *Flatland, a Romance of Many Dimensions*, Princeton University Press, Princeton, NJ, 1991.

[2] T. F. Banchoff, *Beyond the Third Dimension: Geometry, Computer Graphics, and Higher Dimensions*, Scientific American Library, W. H. Freeman & Co., New York, 1990.

[3] ———, "Dimension", in *On the Shoulders of Giants*, ed. Lynn Arthur Steen, MSEB National Research Council, National Academy Press, Washington, D.C., 1990.

[4] T. F. Banchoff and D. P. Cervone, "Cornucopia", cover image, *Focus*, March, 2000.

[5] A. K. Dewdney, Four-Dimensional Dementia, *Scientific American*, April, 1986.

[6] M. Kaku, *Hyperspace*, March, 1995.

[7] M. L'Engle, *A Wrinkle in Time*, Dell Publishing Co., New York, NY, 1962.

[8] J. Weeks, *Shape of Space*, Monographs in Pure and Applied Mathematics, Marcell Dekker, Inc., New York, 1985; see ⟨`URL::http://www.geom.umn.edu/locate/SOS`⟩.

[9] Pixar Animation Studios, *Geri's Game*, animated short film, 1997.

[10] Disney Animation Studios, *Toy Story*, animated feature film, 1999.

[11] The MAM2000 web site, ⟨`URL::http://mam2000.mathforum.com/`⟩.

[12] The Math Forum, ⟨`URL::http://www.mathformum.com/`⟩.

[13] Equations for Cornucopia, ⟨`URL::http://mam2000.mathforum.com/612/dimension/cone/equations.html`⟩.

Thomas F. Banchoff
Department of Mathematics
Brown University
Providence, RI 02912

E-mail address: `thomas_banchoff@brown.edu`

Davide P. Cervone
Department of Mathematics
Union College
Schenectady, NY 12308

E-mail address: `dpvc@union.edu`